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PREFACE.

It is to be deeply regretted that, although Mensuration is both an interesting and useful study, it has hitherto been almost entirely excluded from the sphere of elementary education; and that many of the teachers themselves, in our elementary schools, are unacquainted with it. The reason of this general neglect of so important a subject is, no doubt, mainly that it has been the custom of our mathematical teachers to postpone it until their pupils have such an acquaintance with Trigonometry, Geometry, and Algebra, as will enable them readily to understand and apply general and distinctly scientific proofs of the rules which are to be employed. This postponement, however, in the case of elementary schools, has necessarily amounted to banishment of the subject altogether from the range of teaching; and most of the recent text-books on Mensuration have been designed solely for the learner who has made some advance in the study of Mathematics.

Mensuration may be of great service to thousands of persons who will never know anything either of Trigonometry or Euclid, and for these learners of lower attainment this little book is designed. The author has endeavoured to treat the subject simply, in a series of graduated lessons, such as

can be easily understood by learners whose mathematical knowledge does not go much beyond the simplest parts of Arithmetic, in the hope that it may prove useful to teachers and scholars in our elementary schools, and students in our training colleges, as well as to that interesting class of learners who are endeavouring to teach themselves from books, without the aid of a teacher.

H. L.

CULHAM, *December* 1874.

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MENSURATION.

PART I.

INTRODUCTORY LESSONS.

Lesson I.—Definitions and Explanations.

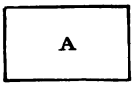
The word Mensuration really means "measuring."—If a farmer wishes to know the exact size of a field, or a builder that of a wall, or if a paperhanger wishes to calculate the cost of papering a room, or a painter that of painting the front of a house; if you had to estimate how many gallons of water a tub or cistern would hold, or how many cubic feet of air there are in a certain room—all these useful operations would have to be performed by mensuration, which is *the art of calculating dimensions of length, area, and volume.*

In studying this subject we shall have to make constant use of certain names, or terms, which some of you doubtless are already familiar with, having had to learn them when commencing the study of Euclid or Geometry. But for the sake of those who may not yet have begun this study we shall insert here a few of the simpler definitions or explanations, and introduce the others, as occasion requires them, farther on in the book.

It ought not to be necessary to remind those who will use this book that accurate definitions are very important, and that therefore, especially in such a study as this, it will be necessary to commit them to memory with great care.

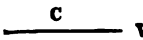
1. A point is that which has no dimensions, *i.e.*, neither length, breadth, nor thickness.

2. A line has length but no breadth.



3. A superficies or surface has length and breadth, as A.

4. A solid body has length, breadth, and thickness, as B.

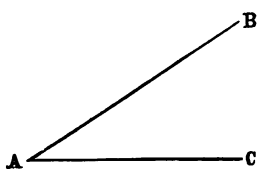


5. A straight line (or right line) is one which lies evenly between its extreme points.

Sometimes it is said, and quite truly, that a straight line is the shortest distance between two points.

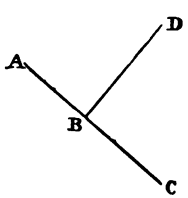
If you look at the water in a pond or cistern, where it is perfectly still, you will notice that its surface is level or horizontal, any straight line on the surface of the water, or parallel to it, is a *horizontal line;* any line which is perfectly upright on it is a *vertical line;* lines which are neither horizontal nor perpendicular are *oblique.*

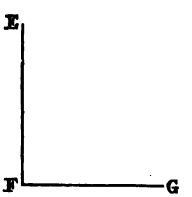
6. An angle is the corner formed by the meeting of two straight lines, as BA and AC.



This would be called *the angle BAC,* three letters being used, andthe letter at the corner always placed in the middle; sometimes for the sake of shortness, and where there would be no ambiguity in such a case, it would be spoken of simply as the *angle at A.*

Angles are said to be of three kinds, *right, acute,* and *obtuse.*





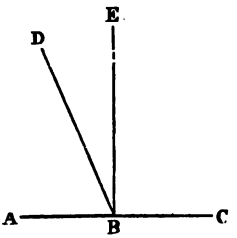
A right angle is formed by one straight line standing erect or perpendicular on another: thus, ABD, DBC, EFG, are three right angles.

Those of you who have begun to learn Euclid will have met with the following further and more perfect explanation of the term "right angle," but it is, perhaps, a little too difficult for beginners:—

When a straight line standing on another straight line makes the adjacent angles equal to one another, each of these angles is called a right angle.

An acute angle is less than a right angle, as ABD, DBE.

An obtuse angle is greater than a right angle, as DBC.



LESSON II.—TABLES OF MEASUREMENT.

In mensuration, we have to measure lines, surfaces, and solids, and to enable us to do this, and to express the result in words which other people will readily and accurately understand, we must learn the following tables of *linear*, *superficial*, and *solid* measure, or (as they are more commonly called) long, square, and cubical measure.

I.—LONG MEASURE.

12 Inches	- -	1 Foot.
3 Feet	- - -	1 Yard.
5½ Yards	- - -	1 Pole (rod or perch).
40 Poles	- - -	1 Furlong.
8 Furlongs	- -	1 Mile (1760 yards).
3 Miles	- - -	1 League.

LAND MEASURE.

7·92 Inches	- -	1 Link.
100 Links	- -	1 Chain.
22 Yards	- - -	1 Chain.
10 Chains	- -	1 Furlong.

II.—SQUARE MEASURE.

144	Square inches - -	1 Square foot.
9	Square feet - - -	1 Square yard.
30¼	Square yards - -	1 Square perch (pole or rod).
40	Perches - - -	1 Rood.
4	Roods or 10 square chains	1 Acre.
640	Acres - - -	1 Square mile.

Note.—The name *acre* always, and the name *perch* nearly always, are restricted to *square* measure.

III.—Solid Measure.

1728 Cubic inches - - 1 Cubic foot.
27 Cubic feet - - - 1 Cubic yard.
1 Gallon contains 277·274 cubic inches.
1 Gallon of distilled water weighs 10 lbs.
1 Cubic inch of cast-iron weighs ¼ lb., nearly.

It is very important to have absolutely perfect examples of all these measurements, so that everybody can refer to them if he pleases as the standards of accuracy. Hence in every town such standards are provided by the authorities, and are used as tests in case of any dispute.

But, as you all have doubtless learned already in that part of arithmetic which is called *Weights and Measures*, besides these standards of reference there is another method of finding whether weights and measures are correct, for all of them can be deduced by calculation from the length of a seconds' pendulum.

Formerly, however, great confusion was caused by the fact that there were no invariable standards of weight or measure, and such names as *pole*, *stone*, or *ell*, had one meaning in one part of the country, and another meaning in another part. In some parts of England this confusion still remains; thus, although the *statute pole* (*i.e.*, the measure of length called a pole which is the only one legally recognised) is equal to 5½ yards, in Scotland 6½ yards make one pole; and, to make matters still worse, in Ireland and the North of England a length of 7 yards, and in Cheshire of 8 yards, is called by the same name.

In the same way the name *ell*, used in cloth measure, has at least four different meanings, according as it happens to refer to English, Flemish, French, or Scotch ells.

Many of the old measures were also useless, because the standards by which they were determined are accidental and variable, *e.g.*, *nail*, *foot*, *hand*, *span*, *barley-corn*, *stone*.

The *chain*, which is a very important length in land measuring, will be remembered more easily if you keep in mind that in a game of cricket the wickets are pitched just one chain apart, *i.e.*, 22 yards. This is generally called Gunter's chain, because its use was first adopted by a man named Gunter.

In France they have a system of measurement called the *metric system*, which is much better and simpler than ours. You will find a full account of this very interesting and perfect system in most books on arithmetic, so that it is not necessary to explain it here. But you must remember that instead of the *yard* which we use in most English measurements of length, the French use the *metre*, which is a little longer than a yard, and contains 39·37,079 inches.

IV.—METRIC MEASURE.

				yds.	ft.	in.
1 Myriametre	=	10,000 metres	=	10,936	0	11·9
1 Kilometre	=	1000 ,,	=	1093	1	10·79
1 Hectometre	=	100 ,,	=	109	1	1·08
1 Dekametre	=	10 ,,	=	10	2	9·71
1 Metre			=	1	0	3·37
1 Decimetre	=	$\frac{1}{10}$,,	=			3·94
1 Centimetre	=	$\frac{1}{100}$,,	=			·39
1 Millimetre	=	$\frac{1}{1000}$,,	=			·04

There are two other names which are sometimes used in French mensuration, viz., *are* and *stere*. The *are* is a square, each side of which measures 10 metres, and which therefore contains 100 square metres. The stere is a cubic metre. Instead of using these two names, the French generally speak of square and cubic metres, which is much simpler.

In order to make you acquainted with the French metric system, a few examples will be interspersed at various places among the other exercises in this book.

EXERCISES (A).

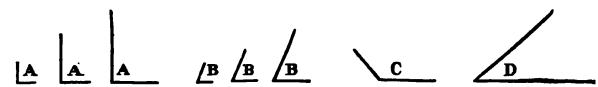
1. Reduce 26 yds. 2 qrs. to nails.*
2. ,, 46 miles 4 fur. 438 yds. to feet.
3. How many acres in 78,235 sq. poles?
4. ,, ,, poles in 469 ac. 3 roods 14 poles?
5. ,, ,, yards in 4348 nails?
6. ,, ,, miles in 147,852 inches
7. ,, ,, English ells in 14,250 yards.
8. Reduce 78,254 Flemish ells to yards.
9. ,, 2800 poles to acres.
10. ,, 489 acres 14 poles to poles.
11. ,, 391,256 cubical inches of cubical yards.
12. ,, 152 ac. 3 ro. 38 poles to poles.
13. ,, 71 miles 1 fur. 6 po. $2\frac{1}{2}$ yds. to yards

* The *nail* is used in cloth measure, and contains $2\frac{1}{4}$ inches.

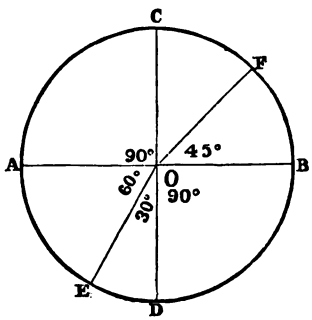
14. Reduce 166 fur. 33 poles 3 yds. to feet.
15. ,, 17 po. 2 yds. 2 ft. 8 in. to inches.
16. ,, 61 miles 7 fu. 19 po. 2 yds. 7 in. to inches.
17. If I walk 2 miles 88 yards in 40 minutes, how long will it take me to walk 390 miles.
18. Multiply 399 ac. 1 ro. 18 po. 27 yds. 8 ft. 36 inches by 98.
19. Divide 76 ac. 0 ro. 4 po. 6 yds. 0 ft. 112 in. by 31.
20. ,, 3687 c. yds. 1684 c. in. by 264.

Lesson III.—Measurement of Angles.

It has been already explained that a right angle is greater than an acute angle, and less than an obtuse angle. It must be carefully remembered that the size of an angle is not at all dependent upon the length of the two lines which make it, but upon the *sharpness* or *bluntness* (*i.e.*, the *acuteness* or *obtuseness*) of the points they make at meeting. Thus, all the right angles marked A are of the same size, and all the angles marked B; while the angle C is much greater than that marked D.



All angles are measured by the number of *degrees* which they contain. Every circle is divided into 360 equal parts, each of which is called a degree. If two diameters of a circle cross each other at right angles, as in the figure, they make four angles of 90 degrees. Every angle which contains 90° is a right angle; half of a right angle (BOF) is an angle of 45°, and a third of one (EOD) is an angle of 30°.



The *right* angle is so called because it is the *standard angle*

by which we measure other angles, just as the straight line is often called a *right line* because it is the standard by which we measure and test all other lines.

Again, it is usual to divide every degree (°) into sixty equal parts called *minutes* (′); and each minute into sixty equal parts called *seconds* (″). In writing, instead of the names we use the symbols, which are marked above in brackets, so that 27° 42′ 23″ is only a short way of writing down 27 *degrees*, 42 *minutes*, 23 *seconds*.

PART II.

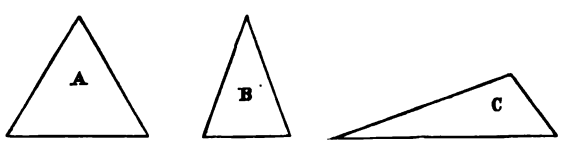
MENSURATION OF LINES AND SURFACES.

Lesson IV.—Right-Angled Triangles.

A TRIANGLE is said to be a *rectilinear* figure, *i.e.*, made up of *right lines* or straight lines. It is called a *triangle* because it has three angles, or (which is exactly the same thing) three sides.

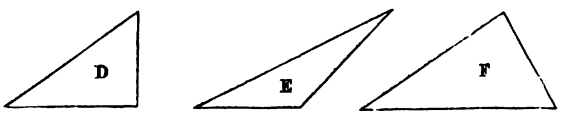
In the same way four-sided figures, of which we shall speak a little farther on, are called *quadrangles*.

Now there is a great deal which is very important and interesting about every triangle, as you have either already begun to learn, or will have to learn shortly when you commence Euclid and Trigonometry. You can never know very much of mensuration or of land-surveying (which is only one application of mensuration to practical purposes), until you have learned thoroughly the properties of triangles. We shall not, however, attempt to treat of this subject here, but shall simply explain a few names which are commonly used in reference to different triangles, and then point out to you a very important truth in reference to every right-angled triangle.



Triangles are sometimes classified according to the comparative lengths of their *sides*, and are then said to be either **equilateral**, *i.e.*, having all three sides equal (as A), or **isosceles**, *i.e.*, having two sides equal (as B), or **scalene**, *i.e.*, having all three sides unequal (as C).

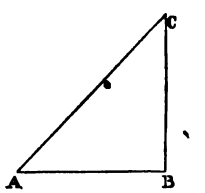
Triangles are also often classified according to their *angles*. If a triangle contains *one right angle* (as D) it is called a **right-angled** triangle; if it contains *one obtuse angle* (as E) it is said to be **obtuse-angled**; but if *all three angles are acute* (as F) it is an **acute-angled** triangle.



Some of you will have learned already that it is impossible for any triangle to have more than one right angle or one obtuse angle, because the three angles of every triangle, when taken all together, make up exactly two right angles; so that every right-angled and every obtuse-angled triangle must have two acute angles, while every acute-angled triangle has three.

Now, the sides of a right-angled triangle are known by special names, the *base* (AB) and the *perpendicular* (BC), which contain the right angle, and the *hypothenuse* (AC), which is opposite to the right angle.

It is an important truth which some of you have learned already in the first book of Euclid, and which will help you to solve many problems in mensuration, that *in every right-angled triangle the square upon the hypothenuse is equal to the sum of the squares upon the base and perpendicular.*



From this we get two valuable rules which enable us, when we have given any two sides of a right-angled triangle, to find the other side:—

1. To find the hypothenuse, add the square of the base to that of the perpendicular, and take the square root.

2. To find either the base or the perpendicular, subtract the square of the side given from that of the hypothenuse, and take the square root.

Example 1.—Find the perpendicular of a right-angled triangle whose base is 56 feet and hypothenuse 65 feet.

Square of hypothenuse, $= 65^2 = 4225.$
Square of base, $= 56^2 = 3136.$
Therefore square of perpendicular, $= 4225 - 3136 = 1089.$
and perpendicular, $= \sqrt{1089} = 33.$
Answer, 33 *feet.*

Example 2.—Find the hypothenuse of a right-angled triangle whose base is 85 feet and perpendicular 132 feet.

Square of base, $= 85^2 = 7225.$
Square of perpendicular, $= 132^2 = 17424.$
Therefore square of hypothenuse, $= 7225 + 17424.$
$= 24649.$
and hypothenuse $= \sqrt{24649}.$
$= 157.$
Answer, 157 *feet.*

Exercises (B).

1. In an isosceles triangle the angle at the vertex is 25° 15′ 16″, what is each angle at the base?
2. A certain right-angled triangle has one of its acute angles double of the other; find the angles.
3. In an isosceles triangle the angle at the vertex is double each angle at the base; find the angles.
4. Find hypothenuse when base = 51 yds. and perpendicular 67 yds.
5. ,, ,, = 82 ft. ,, 106 ft.
6. ,, ,, = 21 ft. 8 in. ,, 5 ft. 9 in.
7. ,, ,, = 38 ft. 6 in. ,, 86 ft. 8 in.
8. ,, ,, = 31 ft. 6 in. ,, 56 ft. 8 in.
9. ,, ,, = 105 ft. ,, 88 ft.
10. ,, ,, = 16 ft. 8 in. ,, 103 ft. 6 in.
11. ,, ,, = 12 ft. 5 in. ,, 6 ft. 3 in.
12. Find base when perpendicular = 46 yds. and hypothenuse 530 yds.
13. ,, ,, = 117 ft. ,, 125 ft.
14. A field is in the form of a right angle, the base being 1462 yds., and hypothenuse 2138 yds.; how many yards of fencing will be required to go round the field?
15. The side of a square is 1000 feet; what is the length of its diagonal?
16. What is the height of an equilateral triangle, each of whose sides measures 20 feet.
17. The base of an isosceles triangle measures 50 ft., and each side 65 feet; find the perpendicular.
18. A field is in the form of a right-angled triangle, the base being 1584 yds., and the perpendicular 2350 yds.; find the cost of fencing it all round, at 10d. per yard.
19. In a similar field the hypothenuse is 1010 ft., and the base 434 ft.; find the cost of fencing it, at 9½d. per yard.

20. In a similar field, the base is 720 ft., and the hypothenuse 962 ft.; find the cost of fencing, at 10½d. per yard.

21. A rope, 76½ feet long, will just reach from one side of a street to the top of a house 67½ ft. high, exactly on the opposite side; how wide is the street?

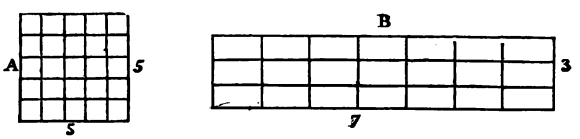
22. A man who is carrying down a street a ladder 50 ft. long, rests it upon the road in such a position that it will exactly reach a window 28 ft. high on one side, or another window 36 ft. high on the other side; find the width of the street.

23. The diagonal of a square is 20 yards; find the length of each side.

24. A perpendicular rock 103 ft. high stands close to a river whose breadth is 302·97 ft; find the length of the cord which will exactly reach from the top of the rock to the opposite bank of the river.

LESSON V.—THE SQUARE AND RECTANGLE.

The square and rectangle both belong to that class of figures previously mentioned, and called *quadrangles;* they are both contained by four straight lines, and all the angles in each are right angles; but in the square all the sides are equal (A), while in the rectangle only the opposite sides are equal (B).



To find the area of a square or rectangle, multiply two adjacent sides together.

Thus, the area of A is $5 \times 5 = 25$; and that of B is $7 \times 3 = 21$. The correctness of this rule may easily be tested experimentally; the two figures above are divided out into a number of small squares, each containing exactly one square unit, and there are twenty-five of these in the square A, and twenty-one in the rectangle B.

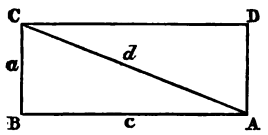
This simple rule lies at the basis of all rules in mensuration, used

for measuring surfaces. Whenever a surface (or *area*) has to be measured, *we always multiply two numbers together*, just as in calculating *volume* or solidity we always multiply *three* numbers together, and then modify the result by introducing another factor so as to get the correct answer. Thus, in one sense, all areas are calculated in reference to the square, and then corrected; and all volumes are calculated in reference to the cube, and then corrected.

What was learnt in the previous lesson (IV.) will enable us now to solve two other interesting problems in connection with squares and rectangles.

1. *To find the area of a rectangle when one side and the diagonal are given—*

For example, in the figure, let us suppose that instead of having given the length of AB, BC (which, for shortness, we will call respectively c and a, from the letters placed at the opposite angles), we have given only a and d. Now the area can be found in no other way than by multiplying a and c, so that we must begin by ascertaining the length of the latter. Now by the preceding lesson we know that

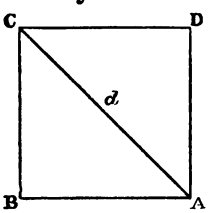


$$c = \sqrt{d^2 - a^2}.$$

And area, $= ac.$

$$= a\sqrt{d^2 - a^2}$$

2. *To find the area of a square when its diagonal is given.*



Let us, as before, call the diagonal d. Now the area of the square is AB^2, or (which is exactly the same), AB × BC; and whatever we do to find the area will simply amount to finding the length of one side and then squaring it. But the preceding lesson has enabled us to find the length of one side, for

$$AB^2 + BC^2 = d^2$$

i.e., $$AB^2 + AB^2 = d^2$$

$$2\,AB^2 = d^2$$

$$AB^2 = \frac{d^2}{2}.$$

But AB^2 is the area of the square, so that we see that to find the area we simply *take half the square of the diagonal.*

Exercises (C).

1. How many square yards of paper will be required for papering a wall 23 ft. long and 14 ft. 6 in. high?
2. A table is $7\frac{3}{4}$ ft. long and $3\frac{1}{2}$ ft. broad; what is its area?
3. A rectangle is 9 in. wide and contains exactly a square yard; find its length.
4. Find the length of a rectangle whose area is 540 sq. ft. and breadth 4 yds.
5. The diagonal of a square is $9\frac{3}{4}$ ft.; find its area.
6. What is the area of a rectangular table 5 ft. 4 in. long and 4 ft. 6 in. broad?
7. A rectangle contains 5 sq. ft. and is 10 inches broad; find its length.
8. A room is 23 ft. $10\frac{1}{2}$ in. long and 25 ft. broad; how much will it cost to cover it with carpet, at 4s. 10d. a square yard?
9. Each side of a square field is 693 ft.; find its area.
10. Find the side of a square field which contains $6\frac{1}{4}$ acres.
11. The diagonals of two square court yards are respectively 110 yds. and 1112 yds.; find the area of each.
12. Find the area of a garden $20\frac{3}{4}$ ft. long and $15\frac{1}{2}$ ft. broad.
13. A certain road is $1\frac{1}{2}$ miles long and 75 ft. broad; what was the cost of its construction at one halfpenny a sq. foot?
14. A square field contains exactly 1 acre; find the length of a side.
15. The sides of a rectangle are 30 yds. 2 ft. 8 in. and 9 yds. 0 ft. 9 in.; find its area.
16. Find the area of a square whose side measures 871 links.

Lesson VI.—The Rhombus and Rhomboid.

In the preceding lesson we dealt only with *rectangular* quadrangles; we have now to consider two figures which differ from the square and rectangle in one respect, viz., that their angles are not right angles. They are, indeed, like squares and rectangles that have been forcibly wrenched (like a loose slate-frame), from their true form, the rhombus (I) being a distorted square, and the rhomboid (II) a distorted rectangle.

These figures, because their opposite sides are parallel, are

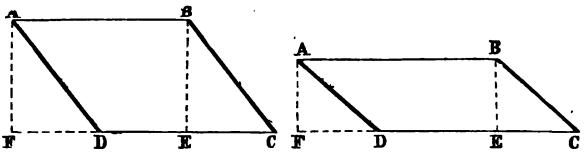
frequently called *parallelograms*, although that name really includes the square and rectangle.

A square has all its sides equal, and all its angles right angles.

A rectangle (or oblong) has its *opposite* sides equal, and all its angles right angles.

A rhombus has all its sides equal, but its angles are not right angles.

A rhomboid has its *opposite* sides equal, but its angles are not right angles.



To find the area of a rhombus or rhomboid, multiply the base by the perpendicular height.

In each of the above figures the area is AB × BE. The reason of this rule is manifest; for in each case the parallelogram ABCD is equal in area to the square or rectangle, ABEF, since they are upon the same base AB, and contained between the same parallels. (Euclid I. 35).

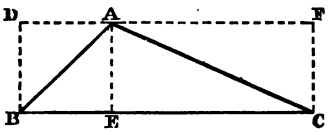
But the area of the latter is in each case AB × BE, and, therefore, the area of the rhombus or the rhomboid is AB × BE.

Exercises (D).

1. The base of a rhomboid measures 2345 links, and the perpendicular height is 2464 links; find the area.
2. The base of a rhomboid measures 385 yds., and its area is 21 ac. 1 r. 30 p.; find the perpendicular height.
3. Find the area of a rhomboid whose length is 40 ft. 6 in. and breadth 12 ft. 9 in.
4. A field in the form of a rhomboid contains 3 ac., and its perpendicular breadth is 198 ft; find its length.
5. What is the rent of a field, at 30s. per acre, which is of rhomboidal form, and 6 chains 20 links long, with perpendicular width 5 chains 14 links?
6. A rhombus is 12·4 ft. long, and its perpendicular breadth is 10·9 ft.; find the area.

Lesson VII.—Triangles.

Case I.—The easiest way by which to find the area of a triangle is by measuring the base and perpendicular height. In the accompanying figure, let us suppose that we know the length of BC and AE.



Now if we multiply these together their product gives the area of the enclosing rectangle DBCF, which we know to be exactly double of the triangle ABC. (Euc. I. 41). The area of which can, therefore, readily be found by dividing by 2.

To find the area of a triangle, multiply half the base by the perpendicular height.

Case II.—The area of a triangle may, however, be determined from its three sides by the following rule:—

From half the sum of the three sides, subtract each side separately; then multiply the half sum and the three remainders together, and the square root of the last product will be the area of the triangle.

For practical purposes this rule may be more conveniently stated in the form of the following formula, where s represents half the sum of the sides:—

$$\text{Area} = s\sqrt{s(s-a)(s-b)(s-c)}$$

Note.—If the triangle is equilateral, all that is necessary is evidently to measure one side. The area may then be calculated from this by the following short and useful rule, *multiply the square of the side by* ·433, *and the product will be the area.* The proof of this rule may be exhibited thus:—

$$\begin{aligned}
\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\
&= \sqrt{s(s-a)(s-a)s-a)} \\
\text{but } s &= \frac{3a}{2} \\
\text{and } s-a &= \frac{a}{2} \\
\therefore \text{area} &= \sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}} \\
&= \frac{a^2\sqrt{3}}{4} \\
&= \frac{a^2\times 1{\cdot}732}{4} \\
&= a^2\times{\cdot}433.
\end{aligned}$$

Exercises (E).

Find the area of each of the following triangles, the base and perpendicular respectively being—

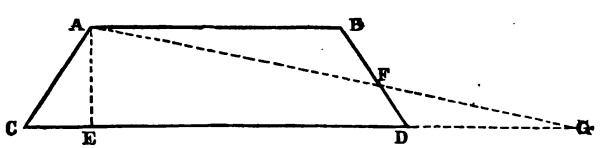
1.	22 ft. 5 in.	and	9ft. 4 in.
2.	3568 links	,,	3178 links.
3.	21·68 ft.	,,	18·92 ft.
4.	12 ft. 6 in.	,,	9 in.
5.	14 ft. 6 in.	,,	4 ft. 9 in.
6.	33·4 chains	,,	7·5 chains.
7.	24 ft.	,,	15 ft. 6 in.
8.	162 ft. 3 in.	,,	32 ft. 5 in.
9.	2750 links	,,	380 links.
10.	1248 links	,,	1890 links.
11.	49 ft. 6 in.	,,	85 ft. 6 in.
12.	20 chains	,,	363·091 links.
13.	850 links	,,	612·5 links.
14.	216 ft. 6 in.	,,	8 ft. 10½ in.
15.	59 ft. 4 in.	,,	10 ft. 2 in.
16.	21 yds. 1 ft. 10 in.	,,	3 yds. 10 in.

Find the area of each of the triangles, whose sides are respectively as follows:—

17. 16 chains 25 links; 15 chains 70 links; 12 chains 45 links.
18. 1500 links; 1200 links; 1450 links.
19. 404 yds.; 495 yds.; 217 yds.
20. 1432 links; 1264 links; 1346 links.
21. 4 ft. 7 in.; 15 ft. 3 in.; 11 ft. 4 in.
22. 10·75 chains; 7·5 chains; 8·25 chains.
23. 29 yds.; 52 yds.; 69 yds.
24. 57 yds.; 60 yds.; 111 yds.
25. 143 yds.; 296 yds.; 375 yds.
26. 164 yds.; 225 yds.; 349 yds.

Lesson VIII.—The Trapezoid.

A trapezoid is a four-sided figure having two opposite sides parallel.



To find the area of a trapezoid multiply half the sum of the parallel sides by the perpendicular distance between them.

The reasonableness of this rule may be demonstrated without any strictly mathematical investigation, by remembering what has been said previously—that we calculate all areas by reference to the rectangle; for if we multiply the longer side CD by the perpendicular distance, the product will obviously be greater than the area of the trapezoid; while if we multiply the perpendicular distance by the shorter side AB, the product will be too little; hence we find a side of mean or average length by dividing the sum of AB and CD by 2, and the product of this and the perpendicular distance is the exact area.

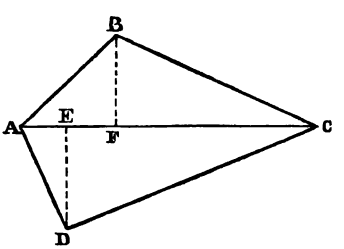
But the rule may be proved mathematically thus: bisect BD in F; join AF, and produce it to meet CD produced in G. Now it is evident that the triangle ABF is similar and equal to FDG, and that AB is equal to DG; so that the trapezoid is exactly equal in area to the triangle ACG.

$$\begin{aligned}\text{area of triangle ACG} &= \tfrac{1}{2}\,\text{CG}\,.\,\text{AE}.\\ &= \tfrac{1}{2}\,(\text{CD}+\text{DG})\,\text{AE}.\\ &= \tfrac{1}{2}\,(\text{CD}+\text{AB})\,\text{AE}.\end{aligned}$$

Lesson IX.—The Trapezium.

A trapezium is a four-sided figure, whose opposite sides are not parallel.

This is an irregular figure, and its area is usually found by dividing it as in the figure, into two triangles.



Now, the area of the triangle ABC is

$\frac{1}{2}$ AC . BF,

and area of triangle ACD is

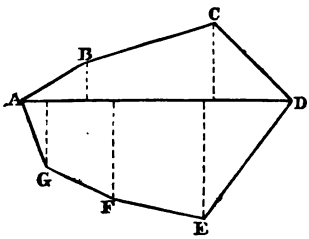
$\frac{1}{2}$ AC . DE;

and, therefore, the area of the trapezium (*i.e.*, of the two triangles together) is

$$\begin{aligned}&\tfrac{1}{2}\,\text{AC}\,.\,\text{BF}+\tfrac{1}{2}\,\text{AC}\,.\,\text{DE}\\ &=\tfrac{1}{2}\,\text{AC}\,(\text{BF}+\text{DE}).\end{aligned}$$

To find the area of a trapezium, multiply half the diagonal by the sum of the perpendiculars.

This is an important and useful rule in all mensuration and land-surveying; when any rectilinear figure has to be measured (especially if it be an irregular one), the most convenient method for calculating its area is to divide it into triangles, or into triangles and trapezoids; and having found the area of each, to take their sum for the entire area. In this way, by means of a simple measuring line and cross-staff, it would be very easy to calculate the area of a field in the irregular shape of the figure ABCDEFG.



Exercises (F).

Find the area of each of the trapezoids whose parallel sides, and the perpendicular distance between them, are respectively as follows:—

1. 243 yds. and 131 yds.; perpendicular distance 352 yds.
2. $12\frac{1}{4}$ ft. and $7\frac{1}{2}$ ft.; ,, 15·4 ft.
3. $12\frac{1}{2}$ ft. and $1\frac{2}{3}$ ft.; ,, 11 in.
4. $18\frac{1}{2}$ ft and $21\frac{1}{4}$ ft.; ,, $8\frac{5}{11}$ ft.
5. 30 ft. 9 in. and 54 ft. 6 in.; ,, 18 ft. 3 in.
6. 684 links and 856 links; ,, 985 links.
7. 46 ft. 10 in. and 28 ft. 4 in.; ,, 53 ft. 6 in.

8. The diagonal of a trapezium is 42 yds. 3 in., and the perpendiculars are 19 yds. 1 ft. 6 in., and 21 yds. 2 ft. 3 in.; find the area.

9. Find the area of a trapezium whose sides, taken in order, are AB=192 yds.; BC=323 yds.; CD=156 yds.; DA=345 yds.; and the diagonal AC=438 yds.

10. The diagonal of a trapezium is 65 yds., and the two perpendiculars 28 yds. and 38·5 yds.; find the area.

11. The diagonal AC of a trapezium is 48·35 chains, and its sides are AB=41·05 chains; BC+37·55 chains; CD=37·75 chains; DA=27·4 chains; find the area.

12. A field in the form of a trapezium contains 16 ac. 3 ro. 8 po.; the diagonal is 16 chains, and the perpendiculars are in the ratio of 14 to 10; find the perpendiculars.

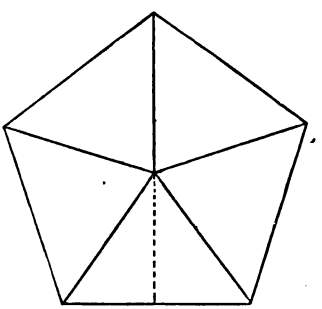
LESSON X.—REGULAR POLYGONS.

A regular polygon is a figure which has all its sides and all its angles equal.

The name *polygon* (meaning *many-cornered*) would seem naturally to include none but figures with five sides or more, and to exclude the triangle and the square. Strictly speaking, however, they are all figures of the same class, and have common properties, and therefore the triangle and square are included in the following table.

Regular polygons are called *pentagons*, *hexagons*, *heptagons*, *octagons*, etc., according as they have 5, 6, 7, or 8, etc., sides.

A polygon may be regarded as divided from the central point into as many triangles as the polygon has sides. A line drawn from this point to the middle point of any one of the sides, will be the radius of the circle inscribed in the polygon, which is equal to the perpendicular height of each triangle. Now the area of each triangle is half the base multiplied by the perpendicular height; and, therefore, the area of the whole polygon may be found by multiplying this by the number of sides.



To find the area of a regular polygon, multiply half the perimeter by the radius of the inscribed circle.

Without knowing the radius of the inscribed circle, the area of a polygon may be found from the length of one of its sides, to the square upon which it bears a constant ratio.

In the following table, the third column gives the ratio which the radius of the inscribed circle bears to the side; and the fourth column gives the ratio of the area of the polygon to the square upon one side.

In calculating the area by the former way, the radius of the inscribed circle must be found by multiplying the length of one side by the number in the third column; by the

latter way, the area may be found at once by squaring the side, and multiplying it by the number in the fourth column.

No. of sides.	Name.	Ratio of radius of inscribed circle to side.	Ratio of area of polygon to square on side.
3	Trigon or equilateral triangle.	·2887	·433
4	Tetragon or square.	·5	1·
5	Pentagon.	·6882	1·7205
6	Hexagon.	·8660	2·5981
7	Heptagon.	1·0383	3·6339
8	Octagon.	1·2071	4·8284
9	Nonagon.	1·3737	6·1818
10	Decagon.	1·5388	7·6942
11	Undecagon.	1·7028	9·3656
12	Duodecagon.	1·866	11·1962

Exercises (G).

1. Each side of a duodecagon measures 93 in.; find its area.
2. What is the cost of painting a heptagonal ceiling at 1s. 6d. per square yard, each side measuring 6 feet.
3. Find the area of a regular pentagon whose side measures 26 yds. 2 ft. 3 in.
4. The side of a regular pentagon measures 61 inches; find the radii of its inscribed and circumscribed circles.
5. The side of a pentagon measures 15 ft. 5 in.; find its area.
6. The side of a heptagon measures 35 in.; find its area.
7. Find the area of a field of octagonal shape, whose sides measure 12 chains each.
8. If the side of a pentagon is 5 yds., what is its area?

Find the area of the following regular polygons:—

9. Decagon with side 2050 links.
10. Pentagon ,, 50 ft.
11. Pentagon ,, 60 ft.
12. Decagon ,, 12 ft.
13. Pentagon ,, 45 ft.
14. Octagon ,, 5 chains.

Lesson XI.—The Circle.

We now pass on from rectilinear figures to those which are called *curvilinear*, being bounded not by straight lines, but by *curved lines*.

A circle is a plane figure contained by one line, which is called the circumference, and is such that all straight lines drawn from its centre to the circumference are equal to one another.

The straight line drawn from the centre of a circle to its circumference is called the *radius*.

The straight line drawn from the circumference through the centre, until it meets the circumference again, is called the *diameter*, and is exactly equal to *twice the radius*.

The diameter divides the circle into two *semicircles*.

The rule for finding the area of a circle may be stated in various ways, all of which, however, are but different ways of stating one and the same thing. One of the simplest of them is as follows:—

To find the area of a circle, multiply half the circumference by the radius.

You will no doubt notice that this is really the same rule as that for finding the area of a regular polygon. The circle may be regarded as a polygon with an innumerable number of sides. A triangle is very unlike a circle, but a square is somewhat nearer to it; a pentagon is nearer still; a hexagon and heptagon still nearer; a duodecagon is very much like a circle, and as the number of sides of the regular polygon increases, the figure becomes more and more circular, until at last the straight sides and the corners are no longer perceptible, and the figure is practically a circle. The rule for finding the area of a circle is only an adaptation of that for finding the area of a polygon, and may be stated in the same terms.

No. of sides.	Perimeter of circumscribed polygon.	Perimeter of inscribed polygon.
4	4	2·82842
8	3·3137	3·06146
16	3·18259	3·12144
32	3·15172	3·13654
64	3·14411	3·14033
128	3·14222	3·14127
256	3·14175	3·14151
512	3·14163	3·14157
1024	3·14160	3·14158
2048	3·1415951	3·1415914
4096	3·1415933	3·1415923
8192	3·1415928	3·1415923

The greater the number of sides which contain a polygon, the more nearly do the inscribed and circumscribed circles coincide with one another. This is exemplified by the foregoing table, which shows in order the ratio to the diameter of the circle of the perimeter of the circumscribed and inscribed polygons, beginning with a polygon of 4 sides, and proceeding by doubling the number of sides up to 8192.

The above table is valuable, because it enables us to determine a very important ratio, viz., that of the circumference of a circle to its diameter. It is evident that the circumference of a circle is greater than that of the inscribed polygon, and less than that of the circumscribed. When the polygon does not contain many sides, the difference between these two circumferences is very perceptible; but as the number of sides is increased, it becomes less and less, until at last it is expressed only in the 7th decimal place, and is only one two-millionth part of the diameter; or, in other words, the exact circumference is less than 3·1415928 times, and more than 3·1415923 times the diameter. This difference is so infinitesimal as to be quite inappreciable, and we may therefore safely say that *the circumference of a circle is* 3·14159 *times the diameter.*

Now, in the mensuration of all circular figures, this number constantly occurs, and is of the greatest importance, so that you must be careful to remember it, and not have to search for it in your book every time it is wanted. But as it would be troublesome to have to write down this number several times in a single calculation, mathematicians are accustomed, for convenience, to represent it by the Greek letter π.

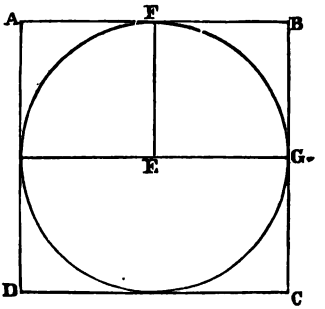
But as the diameter is exactly twice the radius, the circumference of a circle is—radius × 2 × 3·14159, or, as it is generally stated,

$$\textit{circumference of circle} = 2\pi r.$$

The area and the circumference of a circle are dependent upon the length of its radius.

Now the square (ABCD) of the diameter gives us more than the area of the circle, and must therefore be multiplied by a number somewhat less than unity to give it us. This number is really ·7854, or $\frac{\pi}{4}$.

Or, again (which comes to the same thing), if we square the radius, we get the area of EFBG, and it is evident that the area of the circle is not so much as four times this; the truth is that it is exactly 3·14159 times the square of the radius, or, as it is more commonly expressed,



$$\textit{area of circle} = \pi r^2.$$

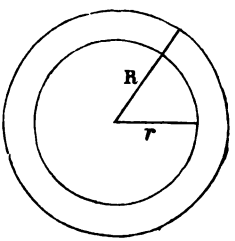
These two latter rules are sometimes stated in words thus:—

To find the area of a circle—

1. Multiply the square of the diameter by ·7854; or,

2. Multiply the square of the radius by 3·14159.

While dealing with the subject of the circle, it will be as well to discuss the method for finding the area of a ring, *i.e.*, of the space included between the circumference of two concentric circles.



It is evident that the area of the ring may be found by subtracting that of the smaller circle from that of the larger. This, however, may be put in a simpler form thus:—

Let R denote the radius of the larger circle,
Let r ,, ,, smaller circle.
Then area of larger circle $= \pi R^2$,
and area of smaller circle $= \pi r^2$;
therefore area of ring $= \pi R^2 - \pi r^2$
$= \pi(R^2 - r^2)$
$= \pi(R+r)\,(R-r)$.

The rule may be expressed in words thus:—

To find the area of a ring, multiply the sum of the radii by their difference, and this product by 3·14159.

Exercises (H).

Find the circumference of each of the circles whose diameters are respectively

1. 28 yds.
2. 120 ft.
3. 24·25 ft.
4. 7912 miles.
5. 3 ft. 9 in.
6. 883220 miles.
7. 7680 miles.
8. 7958 miles.
9. 483 ft.
10. 126 ft.

Find the diameter of each of the following circles whose circumferences are respectively

11. 144 miles.
12. 24 ft. 9 in.
13. 9 ft. 6 in.
14. 6850 miles.
15. 177 ft.
16. 1250 miles.
17. 11 yards.
18. 64 ft.
19. 483 ft.
20. 55 in.

Find the area of each of the circles whose dimensions are as follows:—

21. Diameter 14 links.
22. Circumference 3·14159 ft.
23. ,, half a mile.
24. Radius 39·25068 yds.
25. How many sq. yds. of carpet will be required to cover a circular floor 73·25 ft. across?
26. A coach wheel turns round 250 times in travelling half a mile; what is its diameter?
27. A locomotive engine is travelling at the rate of 60 miles an hour, and the radius of its driving wheel is 3 ft.; how many times does it turn round in a second?
28. To enclose a circular garden there are required 235½ yds. of wire fencing; what is the area of the garden?
29. A circular field contains exactly one acre; find the length of the hedge which encloses it.
30. A circular meadow is 525 ft. in diameter; what would be the cost of making round the outside of it a road 21 ft. 6 in. wide, at 2s. 6d. per square yard?
31. A square is inscribed in a circle whose radius is 10 ft.; find the side of the square.

32. A large circular iron plate, 12 ft. in radius, weighs 15 lb. for every square inch in its surface; find the weight.

33. A circular gravel walk contains exactly 100 sq. yds., and its inner edge is 14 yds. long; what is the length of its outer edge?

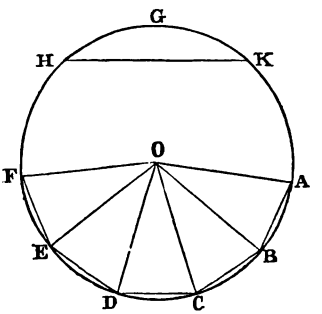
34. The circumferences of two concentric circles are 62·832 ft., and 37·6992 ft.; find the area of the ring enclosed between them.

35. The inner diameter of a circular ring is 6 chains 25 links, and its outer diameter 10 chains 35 links; find its area.

36. The radius of a carriage wheel is 1 ft. 9 in.; how many times will it turn round in going 2 miles?

LESSON XII.—SECTORS AND SEGMENTS OF CIRCLES.

A portion (HGK) of the circumference of a circle is called an *arc*. A figure which is bounded by two radii and an arc of a circle is called a *sector*, as AOB, FOE, GFOA. A figure which is bounded by a straight line drawn through a circle, and the part of the circumference which it cuts off is called a *segment* of the circle, as HKG.



It will be seen that the sector FAO of the circle is something like a triangle, but having a curvilinear instead of a rectilinear base. The arc ABCDEF may, however, be regarded as composed of a great number of very short straight lines, and the whole sector, as thus composed, of a great number of triangles. Now the area of each triangle is half the product of the base and the perpendicular height; but if we regard the number of triangles as very great, the bases would practically coincide with the arc, and the perpendicular with the radii. We may, therefore, regard the whole arc as corresponding to the base of the triangle, and the radius of the circle as representing the height. The rule may then be stated thus:—

To find the area of the sector of a circle, multiply half the length of the arc by the radius.

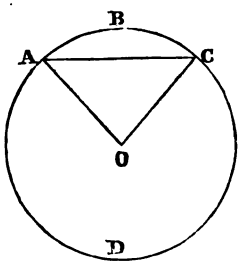
A more satisfactory proof of this rule, which some of you perhaps will understand, is as follows:

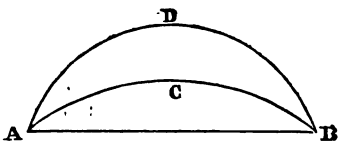
Sectors of a circle are to each other as the arcs on which they stand. (*Euclid VI.*, 33).

$\therefore$ Sector : quadrant :: arc of sector : $\frac{1}{4}$ circumference.

i.e. Sector : $\frac{\pi r^2}{4}$ :: arc : $\frac{\pi r}{2}$

$$\text{Sector} = \frac{\frac{\text{arc} \times \pi r^2}{4}}{\frac{\pi r}{2}} = \frac{\text{arc} \times r}{2}$$





The rule for finding the area of a *segment* may easily be deduced from this:

If the segment is less than a semicircle, ABC, its area may be found by subtracting from the sector AOCB the area of the triangle AOC; and if the segment be greater than a semicircle, as ADC, its area may be formed by adding to the area of the sector ADCO that of the triangle AOC.

In the same way the area of a *lune* or crescent (*i.e.*, the space included between the arcs of two eccentric circles, as ADBC) is evidently the difference between the areas of the segments ADB and ACB.

Eccentric circles are those which have not the same centre.

Exercises (I).

1. The diameter of a circle is 3 yds., and the arc of a sector of it contains 96 degrees; find the area of the sector.
2. Find the area of a sector the radius of which is 10 ft. and the arc 20 ft.
3. The radius of a sector is 8 yds. 1 ft., andi ts arc contains 147° 29′; find its area.

4. A sector greater than a semicircle has arc 14 yds., and the radius $2\frac{1}{2}$ yds.; find its area.

5. The radius of a circle is $32\frac{1}{2}$ yds., and the arc of a sector of it 187° 37′; find the area.

6. A sector of a circle contains 240 sq. ft., and its arc 150°; find the radius.

7. The radius of a circle is 2 yds., and its arc contains 57° 30′.

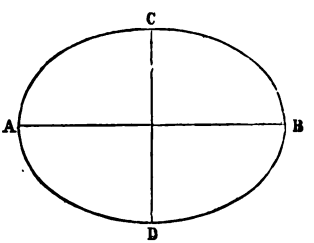
8. The radius of a sector and the length of its arc are respectively $12\frac{1}{2}$ ft. and 16 ft.; find the area.

9. The radius of a sector of a circle is $25\frac{3}{4}$ ft., and the arc contains 218° 54′; find its area.

10. The radius of a quadrant is $27\frac{1}{2}$ ft.; find its area.

LESSON XIII.—THE ELLIPSE.

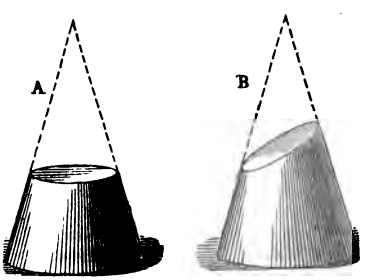
An ellipse, or oval, is like a circle flattened in one direction and elongated in the other.



You will not fully understand this lesson until you have gone farther than at present in your mathematical studies; but it will help you at once if you read carefully what is said about the *cone* in the mensuration of solids farther on in this book.

An ellipse is made by cutting through a cone obliquely.

The base of a cone is circular, and every horizontal section of it (*i.e.*, every section which is parallel to the base, as in fig. A) is circular also, the circle becoming larger the nearer the section is made to the base of the cone. But an oblique section (as in fig. B) forms an ellipse; and the nearer the section is to being perpendicular, the narrower is the ellipse; while the nearer



it is to being horizontal, the broader or more circular is the ellipse.

This will be a convenient place for referring to two other *conic sections*, as they are called. The curve made by a section of a cone, parallell to one of its sides, is a *parabola;* the curve formed by a section which makes a greater angle with the base than the side of the cone makes, is a *hyperbola.*

It would lead to too great a digression to go into the subject more fully, and therefore we shall only add two definitions of the name *ellipse*, which you will understand better when you have gone farther into the subject.

1. *An ellipse is a figure bounded by a regular curve which corresponds to an oblique projection of a circle, or an oblique section of a cone through its opposite sides.*

2. *An ellipse is a curve, such that the sum of the distances of any point in it, from two given points, is equal to a given line.*

The longer diameter AB of an ellipse is called the *major axis*, or the *transverse diameter;* the shorter CD is called the *minor axis*, or the *conjugate diameter.*

The area of an ellipse is found by a method similar to that for finding the area of a circle.

To find the area of an ellipse, multiply the product of the two axes by ·7854; or multiply the product of the semi-axis by 3·14159.

It has been already stated that the area of a circle is equal to the square containing it multiplied by ·7854, or to the square of the radius multiplied by 3·14159. This corresponds to the rule given above, which is only another way of saying that the area of an ellipse is equal to the area of the rectangle containing it multiplied by ·7854, or to the rectangle contained by its longer and shorter radii multiplied by 3·14159.

In other words, an ellipse is equal to a circle whose diameter is a mean proportional between the two axes.

Exercises (J).

1. The axes of an ellipse are 6 ft. and 10 ft.; find its area.
2. The axes of an ellipse are 6 ft. and 7 ft.; find its area.
3. The conjugate diameter of an ellipse is 25 yds., and the transverse diameter 34 yds.; find the area.
4. A plantation in the shape of an ellipse has diameters which measure respectively 110 yds., and 73 yds. 1 ft.; how many stakes will be wanted to fence it if they are placed 10 inches apart?

PART III.

MENSURATION OF SOLIDS.

Lesson XIV.—The Cube and Parallelopiped.

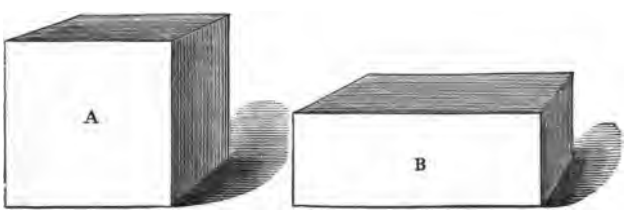
We have already said that a line has only one dimension, viz., length, and that a superficies has two dimensions, viz., length and breadth; hence in calculating area we always proceed by multiplying two dimensions together.

We now pass on to speak of solid figures, which have three dimensions, viz., length, breadth, and thickness, and you will notice that in every case we shall calculate their *volume* or *solidity* by multiplying *three dimensions* together.

Every superficies is bounded by lines; every solid is bounded by superficies.

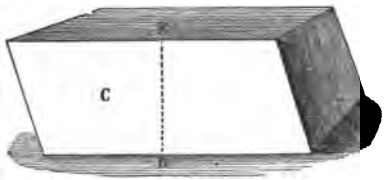
A cube, or solid square, is a solid figure contained by six equal squares (A).

A parallelopiped, or solid rectangle, is a solid figure having six rectangular sides, every opposite two of which are equal and parallel (B).



The above definition really includes only what may be

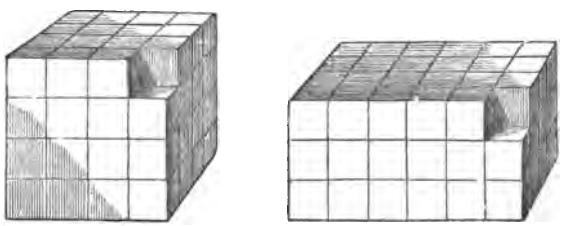
called *rectangular* parallelopipeds; the regular solid C, constructed on a rhombus or rhomboid, is also a parallelopiped, and may be treated by the same rules as the rectangle, only remembering that the thickness of the solid must then be measured by the *perpendicular* distance (D E) of the sides.



A definition of the term as used in this wider sense is:

A parallelopiped is a solid figure contained by six quadrilateral figures, whereof every opposite two are parallel.

To find the volume of a cube or parallelopiped, multiply together its length, breadth, and thickness.



The truth of this may be proved experimentally by actually constructing the figures by means of solid cubical blocks, like the toy bricks of children. But the rule is also evident from the figures given above, which represent a cube and a parallelopiped divided out in this way, one cube being removed from the corner of each to show how they are arranged.

Each side of the cube measures 4 ft., so that by the rule (the length, breadth, and thickness being the same), the volume is $4 \times 4 \times 4 = 64$ cubical ft. But from the figure it is evident that in the square base there are 16 square ft., formed by the square sides of the 16 cubical blocks at the bottom, and upon each of these square feet is a pile of four blocks, so that altogether there are 16×4 blocks, each of one

cubic foot; and the solidity of the whole figure is therefore 64 cubic ft.

In the same way the volume of the parallelopiped may be shown to be $4 \times 6 \times 3 = 72$ cubic ft., for at the base there are 24 square surfaces (4×6), upon each of which is a pile of 3, making altogether $3 \times 24 = 72$; and the volume is 72 cubic ft.

From what has already been said, it is evident that the entire surface of a cube is equal to *six times the area of one of its square sides.*

The surface of a parallelopiped is the sum of the areas of the *six quadrangles which contain it.*

The side of a cube may be found by extracting the cube root of its volume.

Similarly, if the volume of a parallelopiped be divided by the product of any two of its dimensions, the quotient is the other dimension.

Exercises (K).

Find the solid content of the cubes, the side of each being as follows:—

1. 2 ft. 6 in.
2. 7 yds. 1 ft.
3. 7 ft. 6 in.
4. 6 ft. 8 in.
5. 6·5 ft.
6. 12 ft. 9 in.
7. 2 ft. 10 in.
8. 2 ft. 9 in.
9. 3 ft. 4 in.
10. 15 ft. 4 in.
11. 10½ ft.
12. 7 ft. 4 in.

Find the surface of the cubes, the side of each of which is as follows:—

13. 5 ft. 10 in.
14. 10 ft. 3 in.
15. 6 ft. 3 in.
16. 5 ft. 9 in.

Find the solid content of the parallelopiped, whose length, breadth and thickness respectively are as follows:—

17. 70 yds.; 14 yds.; 14½ yds.
18. 24·65 yds.; 1·56 yds.; 2·82 yds.
19. 12 ft. 9 in.; 4 ft. 3 in.; 12 ft. 6 in.
20. 30½ yds.; 28½ ft.; 2 yds.
21. 15 ft.; 1 ft. 9 in.; 1 ft. 9 in.
22. 10 yds.; 5·75 yds.; 3·5 yds.
23. 42 ft. 8 in.; 18 ft. 9 in.; 21 ft. 4 in.
24. 1 yd.; 2½ yds.; 5 ft. 9 in.

25. The surface of a cube is 216 sq. yds.; find the length of its side.

26. The volume of a cubical block of iron is $1\frac{1}{2}$ cubic ft.; how long is its edge.

27. A store closet is 18 ft. long, $8\frac{1}{2}$ ft. broad, and 12 ft. high; what would be the cost of painting it at $9\frac{1}{2}$d. per square yard?

28. A pit is 25 ft. long, 15 ft. broad, and 7. ft deep; what was the cost of digging it at $7\frac{1}{2}$d. per cubic yard?

29. A rectangular log of wood measures at each end 1 ft. 9 in. by 1 ft. 3 in.; what length must be cut off to contain 7 cubic ft.?

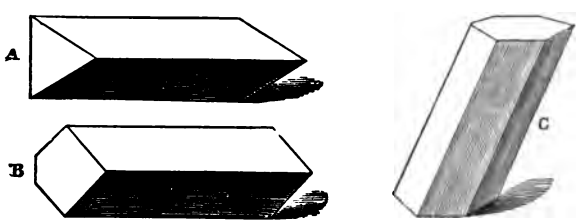
30. A tank is 34 ft. 4 in. long, 4 ft. 3 in. broad, and 6 ft. 5 in. deep; how many gallons of water will it hold?

31. A cistern is $12\frac{1}{2}$ ft. long and 8 ft. wide; if when it is full of water 1000 gallons be drawn off, how much will the surface of the water sink?

32. A tank is 20 yds. long and 14 yds. broad, and will hold 1680 cubic yds. of water; find its depth.

LESSON XV.—THE PRISM.

A prism is a solid whose sides are parallelograms, and its ends two similar and parallel plane figures.



If the sides of a prism are at right angles to its ends (A and B), it is a rectangular prism, and all its sides are rectangles; if the sides are not at right angles to the ends it is an oblique prism (C), and all its sides, except perhaps two, are in the form of a rhombus or rhomboid.

Prisms are generally named from the form of their ends; thus, one which has triangular ends (A) is a *triangular prism;* one which has square ends a *square prism* (*i.e.*, either a cube or parallelopiped); one which has pentagonal ends (B) a pentagonal prism; or hexagonal ends (C) a *hexagonal prism.*

To find the solid content of a prism, multiply the area of one end by the perpendicular height.

We have already said that, in calculating volume, whatever method is adopted amounts practically to multiplying three numbers together; the product of two of the numbers gives us the area of a base, and the product of this and the third number gives the solid content of the body. This is exactly the way in which we have treated the cube and parallelopiped, and it will be readily seen that the same rule holds good for the prism.

Let us suppose that the area of each triangular end of the prism A is exactly 1 square foot; then it follows that if the prism is 1 foot high its solidity is 1 cubic foot, and that, whatever it be, the number of cubic feet in the whole body is equal to the number of linear feet in the height. But whatever may be the area of each end, a certain height cut off from the prism will give a volume of 1 cubic foot, and the volume of the whole prism will be just as many cubic feet as the number of times this height is contained in the whole height of the body.

If the prism be oblique, the area of one end must be multiplied by the *perpendicular* height; for it is manifest that the volume decreases as the perpendicular height is increased, so that the more the sides slope the less is the solid content.

Exercises (L).

1. A triangular prism is 21 ft. long, and of its end one side is 14 ft. long, and the perpendicular let fall on it from the opposite angle is 1 ft. 3 in.; find the solid content.
2. An octagonal prism is 40 ft. long, and each side of its end is 10 ft.; find its volume.
3. The triangular ends of a prism have sides measuring 6, 8, and 10 ft., and the prism is 10 ft. long; find its volume.
4. Find the volume of a square prism, whose length is 41 ft., and one side of its base 15 in.
5. The length of a hexagonal prism is 36 ft., and each side of its base 1 ft. 4 in.; find its solid content.
6. Each side of a pentagonal prism measures 10 ft., and its length is 50 ft.; find its volume.
7. What would be the cost of gilding the entire surface of a pentagonal prism whose length is $32\frac{1}{2}$ in., and each side of its end $6\frac{1}{4}$ in., at $1\frac{1}{4}$d. per cubic inch.

8. A hexagonal prism is 50 ft. 6 in. long, and a straight line across the centre of its end from corner to corner is 30 in.; find its solidity.

9. The volume of a hexagonal prism is 93·532 cubic ft., and each side of its end measures 18 in.; find its length.

10. The volume of a triangular prism is 64·95 cubic ft., and each side of its end measures 30 in.; find its length.

11. A hexagonal prism is 21 ft. long, and contains 1651·564 cubic ft.; find the length of a side of its end.

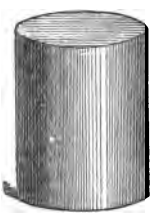
12. A triangular prism, each side of which measures 14 ft. 10 in. is 10 ft. 3 in. high; find its volume.

Lesson XVI.—The Cylinder.

The cylinder may be described as a *circular prism, i.e.*, a prism with circular ends.

It has been already said that a circle is practically a regular polygon with an unlimited number of sides, and this being the case, we have found that the circle is treated in mensuration exactly as if it were a polygon. In the same way the cylinder is really a prism, and the volume or solidity of both is found in the same way, *i.e.*, by multiplying the area of the base by the perpendicular height.

Although, from the above definition and the figure, you will know what a cylinder is, it may be well to add the following more perfect definition:

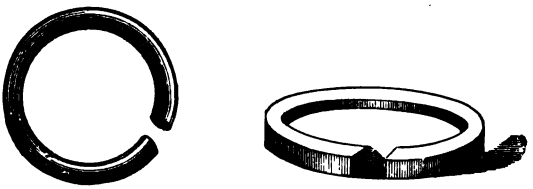


A cylinder is a solid figure described by the revolution of a rectangle about one of its sides which remains fixed.

It is evident that the *whole surface* of a cylinder consists of the two circles forming its ends, and the rectangle forming the sides; the *convex surface* is really a rectangle whose height is the height of the cylinder and base the circumference of the circular end.

The volume of a ring may be easily found by the same rule as that finding the volume of a cylinder. If the ring (A) has a circular section, we may regard it as a cylinder

whose height is equal to the middle line, or mean circumference, of the ring.



If the ring be flat (B) its volume is equal to the difference of the volumes of the outer and inner cylinders of the same height.

Exercises (M).

1. A cylindrical rod of iron is 42 ft. long, and its diameter is 1 ft. 3 in.; find its volume.
2. What is the volume of a piece of the above rod 50 in. long?
3. Find the convex surface of a cylinder whose length is 20 ft., and radius of the base 2 ft.
4. What is the whole surface of a cylinder whose length is 20 ft. 4 in., and radius of base 5 ft. 3 in.?
5. A well is 15 ft. deep, and its diameter is 3 ft. 6 in.; how many cubic yards of earth were taken out in digging it.
6. A cylinder is 20 ft. 8 in. long, and the diameter of its base 11 ft.; find its volume.
7. A circular tunnel shaft is 90 ft. deep and 45 in. in diameter; what was the cost of sinking it at 14s. 6d. per cubic yard?
8. What is the volume of a cylinder 48 ft. long and 6 ft. 8 in. in circumference?

Lesson XVII.—The Pyramid.

A pyramid is a solid, the base of which is any plane figure, and its sides are triangles which meet in a point called the vertex of the pyramid.

A more precise definition which you will find prefixed, among others, to the eleventh book of Euclid, is that *a pyramid is a solid figure contained by planes that are constituted betwixt one plane and one point above it in which they meet.*

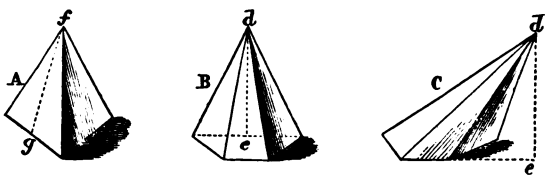
According to the form of its base, a pyramid is either triangular, square, pentagonal, hexagonal, etc.

The *axis* of a pyramid is the line which is drawn from its vertex to the centre of its base.

A *right* pyramid is one the axis of which is perpendicular to the base.

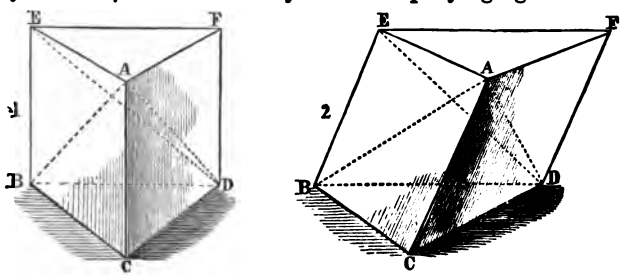
The *altitude* (or perpendicular height) of a pyramid is the perpendicular distance *de* from the vertex to the plane of the base. In right, but not in oblique (or slanting), pyramids this corresponds to the axis of the pyramid.

The *slant height* of a pyramid is the distance *fg* of the vertex from the centre of one of the sides of the base.



The volume of a pyramid is equal to one third of the product of the area of its base and its altitude.

In other words, the volume of a pyramid is exactly one-third of the volume of the prism upon the same base. The correctness of this rule can best be demonstrated practically by a model, as illustrated by the accompanying figures.



Any triangular prism, whether right (as 1), or oblique (as 2), may be divided into three equal and similar pyramids. Thus, if the pyramid ABCD be cut away by a

plane passing through the points A, B, and D, there remains a pyramid with a quadrangular base, BDFE, and apex at A; this again can be divided into two triangular pyramids by a plane passing through A, E, and D, and all three pyramids will then be found to be equal and similar. But the area of the prism is the product of the base and the altitude, and therefore the area of the pyramid is one-third of the product.

The same illustration may be made to apply to a pyramid with a polygonal base, which can always be divided into a number of component triangular pyramids, by planes passing through the apex and the corners of the polygonal base.

Note.—The *surface* of a pyramid is equal to the sum of the areas of the triangles, with or without that of the base, as the case may be.

Exercises (N).

1. The perpendicular height of a pyramid is 20 yds., and each side of its triangular base 6 ft.; find its solidity.
2. A pyramid with a pentagonal base, each side of which measures 4 ft., is 30 ft. high; find its volume.
3. Find the whole surface of a pyramid whose slant height, measured from the apex to the centre of one of the pentagonal sides of the base, is 10 ft., and each side of the base 20 in.
4. A hexagonal pyramid is 10 ft. high, and each side of the base 15 in., and it is of uniform density and weighs 340 lb. per cubic ft.; find its weight.
5. A square pyramid is 15 ft. 9 in. high, and each side of its base 2 ft. 6 in.; find its volume.
6. A triangular pyramid is 19½ ft. high, and the sides of its base are 16½ ft., 14¼ ft. 18⅜ ft.; find its volume.

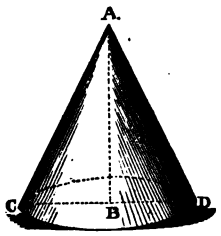
Lesson XVIII.—The Cone.

A cone may be described as a circular pyramid, *i.e.*, a pyramid with a circular base. A more precise definition however is—

A cone is the solid figure described by the revolution of a right-angled triangle (ABC) *about one of the sides* (AB) *containing the right angle, which side remains fixed* (fig. I.)

If the side which remains fixed is equal to the other side con-

taining the right angle, the cone is said to be a right-angled cone, because then a vertical section of the figure always exhibits a right angle (CAD) at the apex, each of the angles BAC and BAD containing 45°.



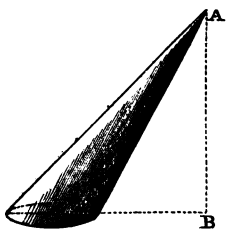
Similarly, if the fixed side be less than the other side, the cone is said to be obtuse-angled; and, if greater, acute-angled; a vertical section of the cones exhibiting respectively an obtuse and an acute angle.

It is obvious that just as the circle is only a kind of polygon (p. 29), so the cone is only a species of pyramid, and may be treated in exactly the same way.

The volume of a cone is equal to one-third of the product of the area of its base and its altitude.

In other words the volume of a cone is exactly equal to one-third of the volume of the cylinder which circumscribes it.

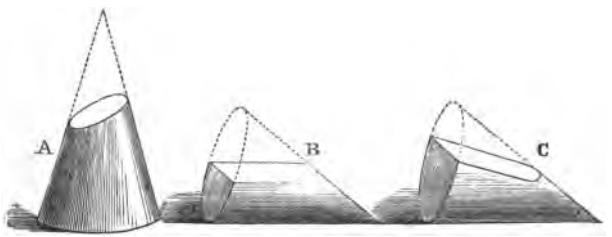
The name *cone* is generally understood to imply only a right or perpendicular cone, as already defined, *i.e.*, one whose apex is exactly over the centre of the base, and whose axis is therefore perpendicular to the base. Sometimes, however, it is used to denote an *oblique* cone (fig. II.), the volume of which is equal to one-third of the product of the area of the base, and the *perpendicular height* AB.



It is evident that the convex surface of a right cone is in the form of a sector of a circle, the radius being the slant height, and the length of the arc the circumference of the base. Hence, to find the area of the convex surface of a cone, *multiply the circumference of the base by half the slant height.*

The cone has been already referred to in the lesson on the ellipse (p. 35) in the mensuration of superficies, where a brief explanation is given of the *conic sections*, viz., the ellipse (A),

parabola (B), and hyperbola (C), which are formed by the intersection in different ways of a plane and a cone.



You will learn more about these figures when you get into more advanced parts of mathematics, which will enable you to investigate all their properties, but here it will be sufficient to say that the interest and importance of these figures is increased when you know that all the planets move round the sun in elliptical orbits, and that the path of a projectile, whether it be a bullet shot from a rifle, a ball from a cannon, or a stone thrown from the hand, is a parabola.

Exercises (O).

1. A cone is 30 ft. high, and the diameter of its base is 1 yard; find its volume.

2. Find the volume of a cone whose height is 8 yds. 1 ft., and circumference of base 6 yds. 2 ft.

3. The diameter of the base of a cone is 9 ft., and its height is equal to the circumference of the base; find its value at 1s. per cubic foot.

4. A cylindrical bottle 4 in. in diameter is filled with wine to the height of 7 in.; how many times will this fill a conical wine glass, 2 in. deep, and $2\frac{1}{2}$ in. in diameter at the top.

5. A cone is 2 ft. high, and its volume is 8 cubic ft.; find the circumference of its base.

6. A cone measures 11 yds. round at the base, and its slant height is $8\frac{3}{4}$ ft.; find its solidity.

7. The slant height of a cone is 13 ft. 5 in., and the diameter of its base is 7 ft. 4 in.; find the cost of plastering its surface at 1s. per square foot.

8. What is the cost of cleaning the curved surface of a cone whose slant height is 18 ft. 8 in., and diameter 13 ft. 4 in., at 1s. per square foot.

9. The slant height of a spire is 45 ft., and the circumference of the base 30 ft.; the cost of painting it is £18, 5s. 7d.; at what rate per square foot is this charged?

Lesson XIX.—The Sphere.

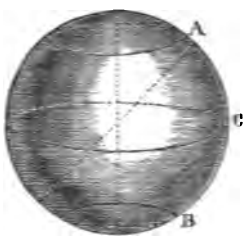
A sphere is a body which is perfectly round in all directions, as a ball or globe.

Of course you are all able to recognise a sphere by its shape the moment you look at it; but, in order to prevent confusion and mistake, it is necessary, as in the figures we have previously examined, to associate the name of the figure with an accurate definition, such as either of the two following:—

1. A sphere is a solid body bounded by one surface, all points of which are equally distant from a point within the body called its centre.

2. A sphere is a body generated by the revolution of a semicircle about its diameter, which remains fixed.

We have already spoken of the figures which are formed when a plane intersects a cone. When a plane intersects a sphere it always makes a circle, which is greater or smaller according as the intersection passes through the centre of the sphere, or is more remote from it. Thus the greatest circle that can be made in a sphere is one formed by a plane passing exactly through its centre; this is called a *great circle* of the sphere.



The *segment* of a sphere is any part of it, A or B, cut off by a plane.

If the plane pass through the centre (C), it will divide the sphere into two equal parts called *hemispheres*.

Now, in speaking of previous figures in this book, we have been careful, as far as possible, to explain to you the reason of the rules which are laid down, and in treating of solid bodies we have spoken first of their volume and afterwards of the area of their surface. In the case of the sphere, however, there is so much that you will not be able at present to understand, that we shall be obliged to give you rules without complete proof, and it will be better also to speak first of the *surface* of a sphere, and afterwards of its *volume*.

The area of the surface of a sphere may be conveniently

found in five different ways, which however, of course, all amount to the same thing, and each of them, as we shall show below, is only a statement in words of the more convenient mathematical expression—

$$\text{surface} = 4\pi r^2.$$

1. *The surface of every sphere is equal to the area of four great circles of the sphere, i.e.*, of four circles, such as already described, formed by a plane passing exactly through the centre of the sphere.

But the radius of such a circle as this is the same as the radius of the sphere itself, and its area is therefore πr^2; and the area of four such circles is $4\pi r^2$.

2. *The surface of every sphere is equal to the area of the circle whose diameter is double the diameter of the sphere.*

But the radius of such a circle is equal to the *diameter* of the sphere, *i.e.* $= 2r$. and

$$\begin{aligned}\text{area} &= \pi \times \text{radius}^2 \\ &= \pi \times (2r)^2 \\ &= \pi \times 4r^2 \\ &= (\text{as before})\ 4\pi r^2\end{aligned}$$

3. *The surface of every sphere is equal to the convex surface of the circumscribed cylinder.*

Now this convex surface is equal to the circumference multiplied by the height; but the circumference is the same as that of the sphere (viz., $2\pi r$), and the height is equal to the diameter (viz. $2r$), so that

$$\begin{aligned}\text{area} &= 2\pi r \times 2r \\ &= (\text{as before})\ 4\pi r^2\end{aligned}$$

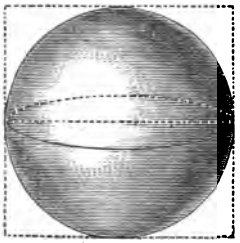
4. *The surface of every sphere is equal to the circumference of the sphere multiplied by its diameter.*

This is exactly the same as the preceding rule:

$$\begin{aligned}\text{circumference} &= 2\pi r \\ \text{diameter} &= 2r \\ \text{surface (as before)} &= 4\pi r^2\end{aligned}$$

5. *The surface of every cylinder is equal to the square of the diameter multiplied by* 3·14159.

But the square of the diameter is $(2r)^2$, which multiplied by $3{\cdot}14{,}159 = 4r^2 \times \pi$ or (as before) $= 4\pi r^2$.



We can now proceed to investigate the method for finding the *volume* of a sphere. The rule for this is usually stated in one of four convenient forms, which, however, as might be expected, are only different ways of stating the same thing, and as will be shown, they are all equivalent to the convenient mathematical expression—

$$\text{volume} = \frac{4}{3}\pi r^3$$

1. Rule I.—The volume of a sphere is equal to the surface of the sphere multiplied by one-third of the radius.

Now we can suppose that a sphere, instead of being bounded by one convex surface is bounded by an infinite number of small faces or plane surfaces; the whole sphere would then consist of an infinite number of pyramids, having these small faces for their bases, their common vertex the centre of the sphere, and their altitude the radius of the sphere. But the volume of a pyramid is equal to the area of its base multiplied by one-third of its height; and therefore the volume of the whole sphere is equal to the sum of the areas of all these small faces (*i.e.*, the whole surface of the sphere), multiplied by one-third of the radius, which

$$= 4\pi r^2 \times \frac{r}{3}$$

$$= \frac{4}{3}\pi r^3$$

2. It will be seen, however, that in this expression the only variable quantity is r^3, and that

$$\frac{4}{3}\pi = \frac{3{\cdot}14159 \times 4}{3} = 4{\cdot}18878.$$

Rule II.—The volume of a sphere is equal to the cube of the radius multiplied by 4·18878.

3. If the diameter of the sphere is given, the foregoing rule may be conveniently stated in another form, thus:

$$\begin{aligned}\text{volume} &= r^3 \times 4{\cdot}18878 \\ &= (2r)^3 \times \frac{4{\cdot}18878}{8} \\ &= \text{diameter}^3 \times {\cdot}52359\end{aligned}$$

Rule III.—The volume of a sphere is equal to the cube of the diameter, multiplied by ·52359.

4. In rules 2 and 3 the volume of the sphere is compared with that of the circumscribing cube, to which it bears a constant ratio; and for rough calculations it is a useful rule that it is equal to a little more than half the volume of the circumscribing cube. We will now compare it to the *circumscribing cylinder*, and we shall find that its volume is always equal to two-thirds of the circumscribing cylinder.

For the volume of the circumscribing cylinder is the area of the circular base, multiplied by the height (*i.e.*, by the diameter); or—

$$\begin{aligned}\text{volume of cylinder} &= \pi r^2 \times 2r \\ &= 2\pi r^3\end{aligned}$$

If we take two-thirds of this we have $\frac{4}{3}\pi r^3$, which is the expression denoting the volume of the sphere.

Rule IV.—The volume of a sphere is equal to two-thirds of the volume of the circumscribing cylinder.

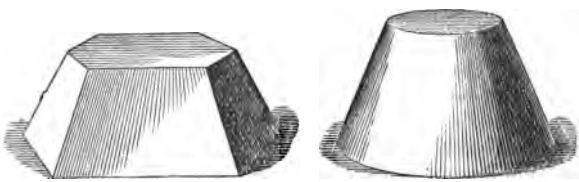
Exercises (P).

1. A globe, whose radius is 28 in., has to be gilded at a cost of 2s. 4d. per square foot; find the cost.
2. The radius of a sphere is $4\frac{3}{4}$ in.; find its solidity.
3. What is the solidity of a spherical shell, whose inner and outer radii are 6 in. and $7\frac{1}{2}$ in.
4. A sphere contains 5946 cubic inches; find its radius.
5. What must be the radius of a sphere which contains exactly 1 cubic yard?
6. The surface of a sphere contains 6734 square ft.; find its radius.
7. The radius of a sphere is $18\frac{3}{4}$ ft.; find its surface.
8. If the cubical content of the earth is taken as 236,855,164,967 cubic ft., what is its circumference, and the area of its surface?
9. A sphere contains 904·708 cubic ft.; find its radius.
10. The circumference of a spherical ball is 4 yds.; what is the area of its surface?

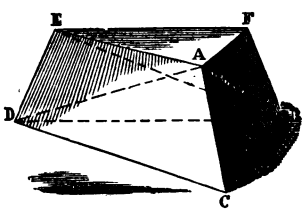
11. The radius of a sphere is 4½ in.; find its solidity.
12. The diameter of a balloon is 6 ft. 2 in.; how many cubic feet of gas will it hold?
13. The radius of a sphere is 1 ft. 4½ in.; what is its surface.
14. The surface of a globe contains 6 sq. yds. 2·7449 ft.; find its diameter.
15. A spherical body contains 2·80957 cub. ft.; what is its circumference?
16. The diameter of the sun is 112 times greater than that of the earth; how many times greater is its volume?

Lesson XX.—Frustrum of Pyramid or Cone.

A frustrum is a solid figure formed by cutting off the top of a pyramid or cone by a plane parallel to the base.



In most instances the solidity of a frustrum may be conveniently found by first finding that of the whole pyramid or cone of which it is a part, and then subtracting the solidity of the piece which is cut off. But in cases where this not convenient, the calculation may be made as follows:—



Let us begin with the frustrum of a triangular pyramid, DCBFE.

Now, if a plane be made to pass through ABD, and another through ABE, the frustrum will be divided into three pyramids, viz., ABCD, whose base is the base of the frustrum, and height the height of the frustrum; ABFE, whose base is the top of the

frustrum AFE, and height the height of the frustrum; and AEDB, whose base is the triangle BDE, and whose vertex is at A. But this last pyramid can be proved to be equal to a pyramid whose height is that of the frustrum, and base a mean proportional between the top and base of the frustrum.

Now these three pyramids taken together make up the whole frustrum, and if A represent the area of the base, a, that of the top, and h the height, then

(1.) Volume of 1st pyramid $= \frac{h}{3} A.$

(2.) ,, 2nd ,, $= \frac{h}{3} a.$

(3.) ,, 3rd ,, $= \frac{h}{3} \sqrt{Aa}.$

$$\therefore \text{Volume of frustrum} = \frac{h}{3} (A + a + \sqrt{Aa}).$$

Now it is evident that a frustrum with a polygonal base may be treated in the same way, and that since the circle is merely a regular polygon with innumerable sides, the frustrum of a cone may also be similarly treated, and the same rule made to hold good for all.

This rule is most convenient for use in the form of the formula given above, but, as in previous cases it may be expressed in words:—

To find the volume of a frustrum of a pyramid or cone, multiply the sum of the areas of the two ends, and of the square root of their product, by one-third the height of the frustrum.

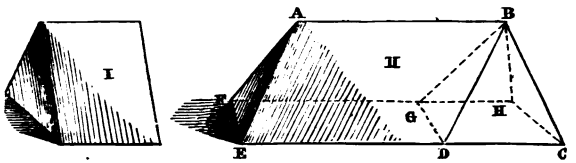
Lesson XXI.—The Wedge.

A wedge is a solid five-sided figure, having a rectangular base, two rectangular or trapezoidal sides meeting in an edge, and two triangular ends.

If the upper edge, or ridge, of the wedge be of the same length as the base (fig. I.), it is evident that the wedge is

only one-half of a rectangular prism, and its volume may be calculated very easily. But if, as in most cases, the edge of the wedge is less than the length (fig. II.), the calculation must be made thus:—

Let us suppose that through B a plane BDG is passed parallel to the end AEF. The whole wedge consists of the solid ABDEFG, and the pyramid BGDCH.



Let us for convenience write the initial letters l for the length, EC, of the base; b for the breadth FE; e for the length of the edge AB; and h for the altitude or perpendicular height.

Then the volume of the solid ABDEFG, which is half the rectangular prism

$$= \text{DE} \times \text{FE} \times \tfrac{1}{2} \text{ height (and DE} = \text{AB)}$$
$$= e \times b \times \tfrac{1}{2} h.$$

Again, the volume of the pyramid BDCHG

$$= \text{CD} \times \text{DG} \times \tfrac{1}{3} \text{ height (and CD} = \text{EC} - \text{AB)}$$
$$= (l - e)\, b \times \tfrac{1}{3} h.$$

Then, total volume of wedge

$$= eb\frac{h}{2} + (l-e)\, b\frac{h}{3}$$
$$= \tfrac{1}{6}\left\{3ebh + 2bh\,(l-e)\right\}$$
$$= \tfrac{1}{6}(2bhl + ebh)$$
$$= \frac{bh}{6}(2l + e).$$

This rule may be expressed in words thus:—

Multiply twice the sum of the length of the base, and

the length of the edge by the product of the breadth of the base, and the height of the wedge; and one-sixth of the last product will be the solidity.

The surface of a wedge may be obtained by finding the area of each surface separately, and then adding them together into one sum.

The frustrum of a wedge is a *prismoid, i.e.*, a solid whose sides are four plane trapezoids, and its ends two right-angled parallelograms which are parallel, but not similar to each other.

It is evident that the frustrum of a wedge is equal to two wedges, whose bases are the thick and thin ends, and common height the distance between them.

By bearing this in mind, you will be able to calculate the volume of haystacks, stone-heaps, colliery waggons, etc.

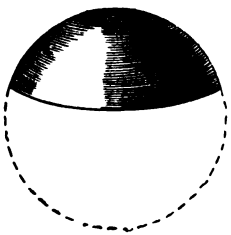
Exercises (Q).

1. A block of marble is in the form of a frustrum of a square pyramid; the side of the smaller end is 1 ft., and that of the larger end 2 ft. 6 in., and the length of the block 16 yds.; find its volume.
2. The sides of the ends of the frustrum of a square pyramid are 2 ft. 6 in. and 3 ft., and their distance apart 5 ft.; find the solidity of the frustrum.
3. Find the solidity of a frustrum of a pyramid, the sides of the octagonal ends being 1 ft. and 8 in. respectively, and their distance apart 3 yds.
4. The girth of a log of timber is 2 ft. 6 in. at the larger end, and 1 ft. 3 in. at the smaller; and its length 25 ft. 9 in.; what is its value at one shilling per cubic foot?
5. The base of a wedge is 5 ft. 9 in. long, and 3 ft. 6 in. broad, the length of its edge 4 ft. 3 in., and the perpendicular height 9 ft.; find its volume.
6. A frustrum of a cone is 25 ft. high, and the circumference of its greater and smaller ends 20 ft. and 10 ft. respectively; find its volume.
7. The sides of the ends of a frustrum of a hexagonal pyramid are 1 ft. and 1 ft. 6 in., and the height of the frustrum 6 ft.; find its solidity.
8. A pentagonal frustrum is 5 ft. high, and the sides of its ends 18 in. and 6 in.; find its volume.

Lesson XXII.—Spherical Segment and Zone.

In this lesson we shall treat, in a simple manner, of two solids which have as yet been passed over; giving short

descriptions of them, and the rules for calculating their volumes.



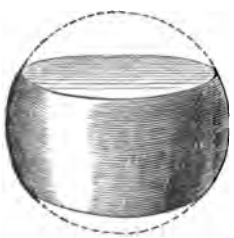
1. *A spherical segment is any part of a sphere cut off by a plane.*

To find the volume of a spherical segment, add together three times the square of the radius of its base and the square of its height; multiply this sum by the height, and the product by ·5236.

This may be more conveniently expressed as a formula, thus—

$$\text{Volume of segment} = \frac{h\pi}{6}(3r^2 + h^2).$$

2. *The zone of a sphere is a part intercepted between two parallel planes.*



If these planes be equally distant from the centre, it is called the middle zone of the sphere.

To find the volume of a spherical zone, add together the square of the radius of each end, and one-third of the square of their distance (*i.e.*, of the height of the zone); multiply the sum by the height of the zone, and this product by 1·5708.

This rule may be expressed as a formula, thus—

$$\text{Volume of zone} = \frac{h\pi}{6}\left\{3(R^2 + r^2) + h^2\right\}$$

Exercises (R).

1. A ball is 3 ft. 6 in. in diameter, and I cut from it a segment 15 in. high; find its volume.
2. A spherical segment is 24·8 in. high, and its diameter 5 ft. 1·2 in.; find its volume.
3. The height of a zone is 1 ft. 3 in., and the diameters of its ends 2 ft. 4 in. and 1 ft. 8 in.; find its volume.
4. The diameters of the ends of a zone are 9 ft. 3 in. and 6 ft. 9 in., and its height is 5 ft. 6 in.; find its volume.

5. From a sphere whose radius is 6 ft., I cut off a segment 4 ft. high; find its volume.

6. A sphere whose radius is 4 ft. 6 in., is divided into three parts of equal height by two parallel planes; find the volume of each part.

LESSON XXIII.—ARTIFICERS' WORK.

In nearly all trades the common standards of measurement are the foot and yard, and calculations may readily be made by practically applying the rules laid down in the preceding lessons of this book. In some trades, however, workmen are accustomed to use terms, and to calculate by rules which we have not as yet explained; and, accordingly, this lesson will be devoted to the explanation of a few such simple terms and rules.

I. In calculations of area for *flooring*, *roofing*, *plastering*, and *tiling*, the standard measure commonly taken is called a "*square*," and contains 100 square ft., *i.e.*, it is the same size as a square, measuring 10 ft. each way.

In many instances it would be inconvenient or dangerous to get upon the top of a house, in order to measure the roof; and, hence, when the *pitch* of the roof is known, the calculation is generally made by measuring only the base of the house. Builders adopt various pitches for roofs, either according to the fancy of the designer, or with regard to the material of which the roof is to be made; but there are three common pitches which have received special names, and are more commonly adopted than any others:—

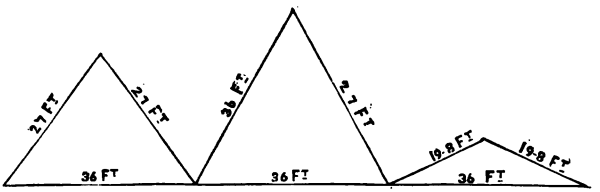
1. *The common pitch*, in which the length of the rafters on each slope of the roof is three-fourths of the breadth of the building, so that the area of the whole roof, including both slopes, is equal to half as much again as the area of the base.

2. *The Gothic pitch*, in which the length of the rafters is equal to the breadth of the building; in this case the area of the whole roof is equal to twice the area of the base.

3. *The pediment pitch*, which is very much flatter, the perpendicular height of the roof being only $\frac{2}{5}$ of the breadth of the building, so that the length of the rafters is nearly $\frac{11}{20}$ of the breadth of the building, and the area of the whole roof is then $\frac{11}{10}$ of the area of the base.

II. By professional engineers the solidity of brickwork is generally calculated by the cubic yard, but workmen com-

monly calculate by what is called a *standard rod*, which consists of a piece of brickwork, a brick-and-a-half thick, and of the area of *one square rod, i.e.*, $30\frac{1}{4}$ square yds., or $272\frac{1}{4}$ square ft. This is by no means a convenient mode of calculating work, and in practice 272 square ft., is commonly taken instead of $272\frac{1}{4}$, which, however, does not simplify it very much.



Common. *Gothic.* *Pediment.*

For convenience, all building bricks are made of one uniform size, viz., $8\frac{1}{2}$ in. in length, 4 in. broad, and $2\frac{1}{2}$ in. thick. But when built in a wall the bricks do not lie close to each other, a layer of mortar encasing them all round; and this layer is supposed to be $\frac{1}{2}$ in. thick, so that in measuring brickwork the dimensions usually taken for each brick are 9 in. long, $4\frac{1}{2}$ in. broad, and 3 in. thick. If 44 bricks be placed flat upon the ground and side by side, with $\frac{1}{2}$ in. of mortar between them they will reach exactly one rod ($5\frac{1}{2}$ ft.); if the wall be then built 66 bricks high, the height also will be exactly one rod; and in the whole square rod 2904 bricks will have been used; but the wall being then only one brick thick, instead of a brick-and-a-half, we must allow for 1452 more bricks, making altogether a total of 4356. Builders, in calculating, usually make reasonable allowance for waste, and reckon that it will take 4500 bricks for every standard rod.

III. Pieces of timber are usually measured by the rules laid down in previous lessons for calculating the dimensions of regular bodies, and for practical purposes the results obtained are tolerably accurate, even when the timber is not perfectly regular in its shape.

The volume of squared (or *four-sided*) timber is generally found by multiplying the length by the mean breadth and

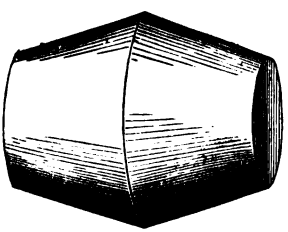
the mean thickness, these last being found either by measuring the actual breadth and thickness in the middle, or else by taking several measurements at various equi-distant points in the length, and dividing the sum of them by the number of measurements.

The volume of round (or *unsquared*) timber is generally calculated by the following rule: *multiply the square of the mean quarter-girt by the length, and the product will be the volume of the timber.* This rule is, however, by no means accurate, and gives a result which is only about three-fourths of the true result, as you will see at once if you consider (p. 30) that the area of the circle representing the middle, or mean, section of the timber is considerably more than the square of a quarter of its circumference. But as this rule is used for calculating the volume of rough pieces of timber, from which a great deal is necessarily lost by cutting and squaring, it gives a result which is generally accepted as fair and reasonable.

IV. The contents of casks are generally measured by a process called *gauging.*

Calculations of this kind are most frequently made by excisemen, for the purpose of ascertaining the amount of duty payable on spirituous liquors kept for sale. Excisemen make their calculations expeditiously by the aid of *gauging instruments*, which, however, it is not necessary to explain or describe now.

Casks vary in shape, and hence no rules can be laid down with regard to estimating their volumes, which are rigidly accurate. But for practical purposes a cask may be supposed to consist of two similar and equal frustrums of a cone, placed together base to base, as in the figure, and the volume may then be calculated by the rule laid down on p. 53.



The method commonly adopted is, however, much simpler than this, and serves tolerably well for all shapes of casks; to apply it three internal measurements must be thrown

in inches, viz., the length of the cask, the diameter at one end, called the *head-diameter* (*d*), and the diameter at the middle, called the *bung-diameter* (D). The rule can be stated in words, thus:—

Add together 39 *times the square of the bung-diameter*, 25 *times the square of the head-diameter, and* 26 *times the product of these diameters; multiply this sum by the length of the cask, and the product by* ·000031473, *and the result will represent very nearly the volume of the cask in gallons.*

Perhaps you will find it less difficult to remember this rule in the form of a formula, thus:—

Number of gallons a cask will hold =

$$l(39R^2+25r^2+26Rr)\times\cdot000031473.$$

If a cask be only partly filled, that portion filled by the liquid is called the *wet ullage;* the portion left unfilled is called the *dry ullage;* and the dimensions of these are expressed respectively in wet and dry inches.

Exercises (S).

1. A wooden partition is 45 ft. 5 in. long, and 8 ft. 2 in. high; find its cost at £13, 10s. per square.
2. The length of a house is 80 ft., and its breadth 35 ft.; find the cost of roofing it, common pitch, at 7s. 6d. per square.
3. A piece of timber is 24 ft. long, and its mean girt is 3 ft. 4 in.; find its solidity.
4. The trunk of a fir tree when felled is 40 ft. long, and the circumference of its ends is 7 ft. and 4 ft.; what would be the common estimate of its value at 1s. per cubic foot.
5. The length of a cask is 30·5 in., and the bung and head diameters 26·5 in. and 23 in. respectively; how many gallons will the cask hold?

PART IV.

LAND-SURVEYING.

Lesson XXIV.—Preliminary Explanations.

One of the most useful and important applications of mensuration is furnished in the measuring of fields, estates, etc., generally called *land-surveying*.

In the preceding parts of this book exercises have been given in which you have been told all necessary dimensions, and have dealt with figures and plans, either ready drawn for you, or which you could easily draw or imagine for yourselves. But you must also learn how to measure actual fields or estates, which you will not find very difficult if you apply with care and caution the rules of mensuration which you have already learned.

To make the explanation of this part of our subject as interesting as possible, we shall suppose that you are to be taken out by some kind friend, who is willing to have the trouble of teaching you experimentally, in order that you may see a surveyor actually go through his work in measuring (or *surveying*, as it is generally called) two or three fields.

But first I will suppose that your friend takes you to the land-surveyor's office, to show you the instruments by which the fields will be measured, and to give a simple preliminary account of their use.

1. You would first notice a number of light poles or rods, such as you could easily carry over your shoulder, 8 or 10 ft. long, with an iron spike at the bottom of each, so that it can be fixed in the ground, and a red or white flag at the top, so that you may more easily see the pole if at a considerable distance.

These poles are called *pickets* or *station staves*, and their use is to stick in the ground, so as to mark certain positions, or guide the surveyor in measuring straight lines.

2. You would then be shown a *Gunter's chain* (so called from the name of the man by whom it was first invented and used), and if you were to stretch out this chain to its full length you would find that it is exactly 22 yds. long (*see p.* 12), *i.e.*, 4 poles, each pole being $5\frac{1}{2}$ yds. An acre of land consists of 10 square chains, or 4840 square yds.

You would notice that the chain consists of a number of links, each about the length of a page of this book, or, to be quite accurate, the exact length of each is 7·92 in. This measurement is called a *link* (*see p.* 11), and the whole chain consists of 100 links.

With the Gunter's chain you would probably notice a small bundle of ten *spikes* or *arrows*, made of stout metal wire, about 10 or 12 in. long, pointed at one end, and each having at the other end a small red or white flag. These arrows are used to stick in the ground so as to mark the measurement of every successive chain, and the use of the flags is simply to make them more conspicuous, for frequently in surveying meadows the grass is so long that it would be difficult to see them unless they were conspicuously marked.

3. One other important instrument which you would be shown consists of a staff or pole of wood, about 5 or 6 ft. high, spiked at the bottom, and having fixed flat on the top a round piece of wood, like a plate. On examining this plate you will find that two straight grooves are cut across it, at right angles to each other, and through the centre. This instrument is called a *cross* or *offset-staff*, and its use is to help in the determination and measurement of perpendiculars, which the measurer does by looking along the cross-grooves of the plate, as will be fully explained in the next lesson.

These simple instruments are quite sufficient for ordinary cases of land-surveying. The various measurements, as they are made, are carefully registered in a *field-book*, the use of which, however, will be explained in the next lesson, when we shall suppose that you are observing a case of actual measurement of a field.

LESSON XXV.

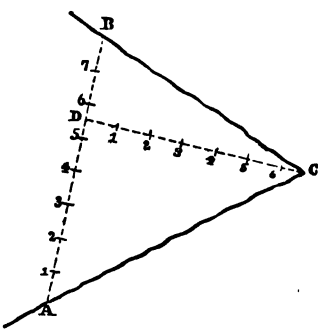
MEASUREMENT OF A TRIANGULAR PIECE OF LAND.

We will suppose that it is required to cut off from one end of a large field a triangular piece, by a straight line of fencing going across the field; and to measure the exact area of the piece which is thus cut off to be sold.

It will be necessary to go down to the field, and make the measurements upon the spot. You must fancy that the land-surveyor is there, with his field-book and pencil ready for making the entries; and that his assistant is with him carrying the pickets, the Gunters' chain and arrows, and the cross-staff.

Now land is very valuable, and it is necessary in land-surveying to be very careful and accurate, for a trifling mistake in the measurements, or in the entries, may make a difference of hundreds of pounds to the persons who buy and sell the land. One very good method for insuring general accuracy is to make, from previous inspection, a rough plan of every field measured, so as to avoid all ambiguity in dealing with the entries in the field book. This method we shall adopt in the present case.

We suppose that the surveyor has been accurately instructed with regard to the piece of the field which he is to measure, and that he knows that the straight line of fencing is to go from A to B. He would take up his first position at A, and at that spot fix a picket. He would then walk across the field and fix another picket at B. It will be necessary also for him to fix upon a spot in the very corner of the field, and, therefore, he would next walk to that corner and fix a third picket at the spot which we have marked in the plan as C.



Now with the Gunters' chain it is easy to measure the line AB, only great care must be taken to count the number

of chains accurately, and to keep always in the straight line between the two points A and B. The surveyor would first go to the picket at A, and place down the end of the Gunters' chain on the ground, putting his heel firmly upon it so as to keep it from slipping; and his assistant would then carry in the one hand the ten arrows or spikes, and in the other hand the other end of the Gunter's chain, and walk straight across the field towards B; and when he has drawn out the whole length of the chain, he would mark the spot by fixing there the first arrow (1) in the ground.

The surveyor, from his stand-point at A, can tell whether the assistant is keeping in the true straight line with B.

The surveyor would then walk with his end of the chain to the first arrow and pick it up, at the same time putting his heel again firmly upon his end of the chain at 1, while the assistant carries the other end towards B till a second length is measured, each carefully watching the other so as to be sure that a perfectly straight direction is preserved, and the assistant then fixing the second arrow in the ground. Both again advance, and the surveyor picks up the second arrow. The same process is continued, the assistant fixing an arrow in the ground at every length of the chain, and the surveyor picking it up as he comes to it, until the end of the line is reached at the point A. The surveyor then, according to the figure, finds that he has seven arrows in his hand, so that the length of the whole line is 7 chains and a short distance over, which, let us suppose, is measured and found to be 86 links.

But you would notice that at a certain time during the measuring of the line AB, the surveyor, who had previously been carrying the offset staff, and keeping a careful look-out on the corner picket at C, would suddenly stop, and, fixing the offset staff firmly in the ground, leave it there, make an entry in his field-book, and then go on as before measuring the line AB. Let us suppose that the point where he so suddenly stopped is 5 chains 24 links from the corner A; this then is the entry which he has made in his field-book.

But why did he stop at this point? Simply because he saw that it is exactly opposite to the corner C, so that the line DC is at right

angles with AB. And how did he know this? You, doubtless, noticed that several times before he finally left the offset staff at D, he tried other places, to see if either of them was exactly opposite C; but when he came to D and tried the staff there, he found that when he looked along one groove at the top, the point B was exactly opposite; when he looked in the opposite direction along the same groove, the point A was opposite; when he looked along the groove at right-angles, the point C was opposite.

The next thing to do is for the assistant and surveyor to come back to the offset-staff at D, and proceed as before to measure the line DC; which, let us suppose, is found to be 6 chains 34 links.

Everything necessary has now been done, and the surveyor can go home and quietly calculate the area of the piece of land, from the entries in his field-book. You noticed that he began writing at the bottom of the page instead of the top, and now you see that, besides a rough plan of the field, similar to that in the figure, he has made this entry:—

to B 786 524 from A.	634 to C.

The meaning of this entry you now understand from the plan, but a surveyor would understand it without a plan, and you must accustom yourself to draw plans from such entries. In the present case the meaning simply is, that a straight base-line goes from A to B, which is 7·86 chains long; and that at a distance of 5·24 chains from A, a perpendicular 6·34 chains long, goes off towards the right to C.

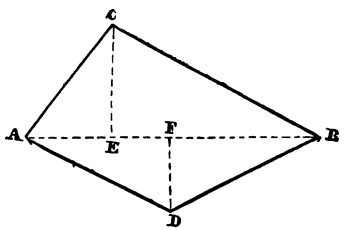
The area of the field may now be calculated thus:—

Area of triangle $= \frac{1}{2}$ base $\times$ perpendicular.
$= 393 \times 634.$
$= 249162$ square links.
$= 2{\cdot}49162$ acres.
$= 2$ acres 1 rood 38 poles 20 yds.

Lesson XXVI.—Measurement of a Four-sided Field.

Having learned in the previous lesson how to calculate the area of any triangular field, we now proceed to inquire into

the best method for dealing with those that are quadrangular, or four-sided. We will suppose that a surveyor has to measure a field of which the accompanying figure is a plan. You see that it is a trapezium, and, therefore, when its dimensions have been accurately ascertained, its area will have to be calculated by the rule already laid down and explained on pages 25 and 26. The dimensions to be first ascertained are *a diagonal, and the two perpendiculars falling upon it.*



Either of the two diagonals, AB and CD, will suit our purpose, but it is always advisable to choose the longer, because then the perpendiculars are shorter; and when you begin to work practically at land-surveying you will find that long perpendiculars are very liable to cause mistakes, on account of the difficulty, even with a good cross-staff, of deciding on the exact point where such a perpendicular will meet the base line. But if the perpendiculars are short, this is easily decided, and as it is comparatively easy to preserve a straight line in measuring, even if it is a very long one, it is the common practice with land-surveyors so to arrange their plans as to make long diagonals and short perpendiculars. For this reason, in the case before us, it will be better to measure the diagonal AB than CD.

The surveyor would first, as before, fix a picket firmly and carefully at each corner of the field, and then with his assistant start from the point A, and measure the line AB, according to the method already described; but during the measurment the surveyor must keep a careful look-out on the pickets at C and D, so as to determine by means of the offset-staff the exact points at which the perpendiculars from them would meet the diagonal.

We continue to speak of the measurers as the *surveyor* and *assistant*, because, as we have seen, the really responsible measurer is the one who comes last and makes the entries in the field-book, the one who goes first being merely his assistant and obeying his directions. In most books, however, you will find these persons called the *follower* and *leader*.

We will suppose that the surveyor finds that on reaching a point (E), 475 links from A, the straight line going to the left to C is at right angles with the diagonal; while a little farther on (F), at a distance of 584 links from A that going on the right to D would meet the diagonal; and that the whole diagonal is 1168 links long. The points E and F would be marked by pickets, and the lines EC and FD measured as before: let us suppose that they are found to be respectively 426 links and 282 links. The entries in the field-book would be as follows:

	to B	
	1168	
	1000	
	584	282 to D
to C 426	475	
	from A	

The area of the field would then be calculated as follows:—

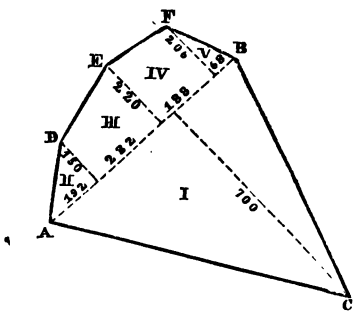
Area of trapezium = ½ diagonal × sum of perpendiculars
= 584 × (426 + 282)
= 584 × 708
= 413472 square links
= 4·13472 acres
= 4 ac. 0 roods 21·5 poles.

Lesson XXVII.—Measurement of Irregular Fields.

We will now deal with one or two cases in which the fields are of irregular shape, and the entries in the field-book more complicated than in the examples previously dealt with. The shape of fields is mostly irregular, and they are bounded, not by straight lines, but by irregularly curved lines. But we have already seen that a curved line may be regarded as consisting of a number of short straight lines, and hence, in land-surveying, if the boundary of a piece of land is irregular the surveyor fixes a number of pickets along it, at such distances from each other that the straight lines joining them practically coincide with the boundary itself, and thus the whole piece of land is divided into a number of triangles, trapezoids, and trapeziums, which may be dealt with as before.

Example I.—Let us suppose that a surveyor has to measure a field which, on inspection, he finds to be in the shape represented by the accompanying figure, bounded by two straight lines (AC, CB), and an irregular line (ADEFB).

He would first of all place pickets at A, B, and C, so as to cut off the triangular portion of the field by a diagonal line, AB. He would then fix pickets at D, E, F, finding that the straight lines AD, DE, EF, FB, practically coincide with the real boundary, so that this remaining portion of the piece of land may be regarded as consisting of two trapezoids and two triangles.



He would then proceed as before, by means of the Gunter's chain and offset-staff, to measure the diagonal and perpendicular. We will suppose that this has been done, and that the following entry is made in the field-book beside the rough plan:—

	to B	
0	680	
to F 206	612	
	488	700 to C
to E 220	424	
to D 150	192	
0	from A	

The entries in the centre column (beginning at the bottom), denote the successive lengths of the various parts of the diagonal line; those on the left hand denote the lengths of the various perpendiculars from the left hand boundary; and the two ciphers denote that, at the two points indicated, the perpendiculars vanish, the boundary line and diagonal meeting in a point.

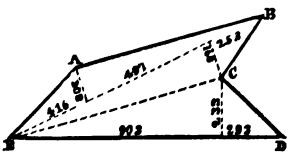
The area of the whole field would then be calculated thus: it consists of the sum of the areas of the figures I., II., III., IV., and V.

Area of triangle	I. =	340×700 =	238000
,, ,,	II. =	96×150 =	14400
,, ,,	V. =	34×206 =	7004
,, trapezoid	III. =	$(110 + 75) \times 232$ =	42920
,, ,,	IV. =	$(110 + 103) \times 188$ =	40044
			342368 sq. links.

= 3·42368 acres
= 3 ac. 1 rood 27·788 poles.

EXAMPLE II.—In this example we will suppose that a field has to be measured whose shape is represented in the accompanying plan. You see that it consists of a triangle DEC, and a trapezium ABCE. Each of these two parts will have to be measured, and there will therefore be two sets of entries in the field-book.

In measuring the triangle, we must ascertain the length of the *base* and of the perpendicular; in the trapezium, the *diagonal* and the two perpendiculars.



It is unnecessary to go through all the details of the measurement, and we shall therefore simply show what would be the entries in the field-book, which by this time you ought to be able to understand without further explanation.

	to B	diagonal EB
	1166	
	1000	
	912	168 to C
to A 308	415	
	from E.	
	to E	
	1196	
	1000	
	293	233 to C
	from D.	

The area would be calculated thus:—

Area of triangle = 598×233 = 139334

,, trapezium = $583 \times (308 + 168)$ = 277508

416842 sq. links.

= 4·16842 acres

= 4 ac. 0 roods 26·947 poles.

In measuring land for sale, when it is surrounded by a hedge and ditch, it is important to remember that, as a general rule, the field to which the hedge belongs includes the ditch also, or, as an equivalent, a distance of three feet outside the hedge. In measuring *crops*, of course this need not be considered.

As an additional insurance of accuracy, surveyors are accustomed to measure one line besides those which are absolutely necessary,

called a *proof-line*, or test-line, and then, when drawing their plans by scale, this line is inserted and measured to see whether it agrees with the measurement actually made with the chain.

Exercises (T).

Draw the plan of each of the fields described in the following notes extracted from a field book, and find the area of each.

1.

	to B	
	1792	
	1000	
	838	562 to C
	from A	

2.

	to B	
	2243	
	2000	
to C 728	1785	
	1000	
	560	624 to D
	from A	

3.

	to B	
to M 360	1314	232 to N
to L 336	1054	
	1000	
to K 100	979	
	854	215 to J
to H 152	759	
	614	312 to G
to F 125	529	
to E 52	334	
	309	234 to D
to C 126	000	O
	from A	

4.

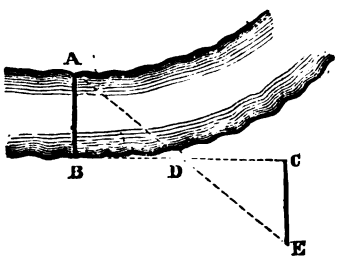
	to B	
	600	
to G 140	560	
to F 150	480	
	470	170 to E
to D 50	380	
	100	150 to C
	from A	

Lesson XXVIII.

To Measure the Breadth of a River.

Sometimes it is necessary to measure a line, the whole or a part of which is inaccessible; the method pursued in such a case always resolves itself into the construction of two similar triangles, one of which can be actually measured with the chain, and has a side either equal to the line in question, or bearing a known ratio to it.

Let us suppose that it is necessary to find the breadth of a river which there is no means of crossing. As you are not able to fix a picket on the opposite side, you must select for observation some conspicuous object on the bank, as a tree or post (A). Then fix a picket exactly opposite to it on the bank of the river upon which you are standing (B); it is evident that the line AB represents the breadth of the river.



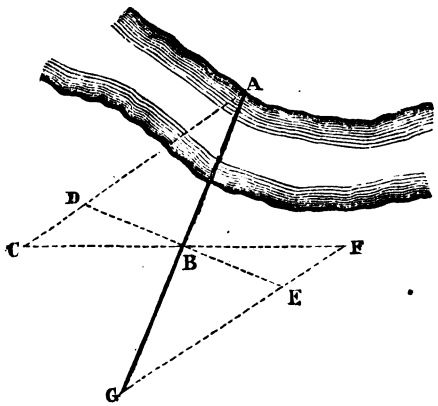
But the point A being inaccessible, the line AB cannot be measured; you can however, by the method of similar triangles, make and measure a line on your own side of the river, exactly equal to it.

By means of the offset staff make a line BC at right angles to AB, and measure off upon it any two equal distances BD and DC, marking them with pickets. Then from D go back in the direction AD, until at E you cross the line drawn from C at right angles to BC. This point can be easily found by using the cross-staff at C. The triangle ABD is similar and equal to the triangle ECD, and EC is equal to AB.

If it is not convenient to take the point B exactly on the edge of the river, it may be taken a little farther back, and the extra distance afterwards deducted.

This method requires the use of the cross-staff, or some other

means of making one line exactly at right angles to another. The measurement may be made however without this, and by using only the chain, as follows:—



Let A be an inaccessible point on the farther bank of a river; choose a point, B, on your own side, either on the edge of the river, or some distance farther back, in which latter case the extra distance must be deducted when the final measurement is made. Draw any convenient line BC, and produce it in the other direction to F, making BF equal to BC. Take any point D in the line CA, and produce the line DB to E, making BE equal to BD. Then having carefully marked all these points, as found, with pickets, go back from B in the direction BG, until the lines AB and FE meet in a point, *i.e.*, until viewed from the same stand-point B and A appear in one straight line, and E and F in one straight line.

Then the triangle GBE is equal and similar to the triangle BAD, and GB is equal to BA.

LESSON XXIX.—EQUIDISTANT ORDINATES.

In common practice, where absolute accuracy is not required, a method of calculation is often applied for finding areas, which gives a result approximately correct, and saves considerable trouble. It is called the method of *equidistant ordinates*.

Let us suppose that a piece of land has to be measured in the shape represented by the boundary line of fig. I. Now the only part of the calculation which is at all difficult arises from the fact that one of the boundary lines is a curve; were it a straight line the figure would be a trapezoid, and its area might be found by a very simple method.

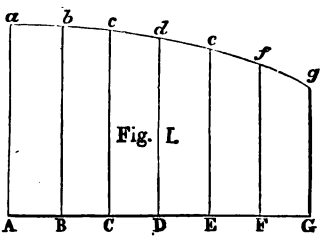


Fig. I.

But in the case before us it is an irregular figure, and the method which most practical measurers would adopt for finding its area is as follows:—

First divide the line AG into any *even* number of equal parts; in the figure it is divided into six: from the several points of division draw the lines B*b*, C*c*, D*d*, E*e*, F*f*, to the curved boundary line, all parallel, and at right angles to the base AG. All these parallel lines are called *ordinates;* and, because they are equally distant from each other, *equidistant ordinates.* Including the two boundary lines A*a* and G*g*, there are seven ordinates; and the reason why we were careful to divide AG into an *even* number of equal parts was to get an *odd* number of ordinates.

The rule which has been found to hold good for calculating approximately the areas of figures may be stated thus:—

Add together { **the first and last ordinates,**
twice the sum of all the other odd ordinates,
four times the sum of all the even ordinates;
and multiply the sum by one-third of the distance between two adjacent ordinates.

Since a formula is more easily remembered than a rule stated in words, the following formula may be used:—

$$\text{area of figure} = \left\{ (a+z)+2\,o+4\,e \right\} \frac{d}{3}$$

a representing the first, and z the last ordinate; o the sum of the *odd*, and e that of the *even* ordinates; and d the common distance between them.

The rule can be best exemplified by a few examples.

EXAMPLE I.—Let us suppose, as in fig. I., that there are seven ordinates, measuring respectively 9·60, 9·38, 9·16, 8·94, 8·72, 8·48, 8·24 chains, and that the ordinates are all 1 chain apart.

9·60	9·16	9·38
8·24	8·72	8·94
17·84	17·88	8·48
	2	26·80
	35·76	4
		107·20

17·84 = *sum of first and last ordinates.*
35·76 = *twice sum of other odd ordinates.*
107·20 = *four times sum of even ordinates.*

3)160·80

53·60 square chains, = approximate area.

EXAMPLE II.—Let us suppose that there are five ordinates, measuring respectively 1·48, 1·66, 1·81, 1·93, 2·04 yards, and that the ordinates are all 2 yards apart.

1·48	1·81	1·66
2·04	2	1·93
3·52	3·62	3·59
		4
		14·36

3.52 = *sum of first and last ordinates.*
3·62 = *twice the other odd ordinates.*
14·36 = *four times sum of even ordinates.*

21·50
2

3)43·00

14·33 square yards = approximate area.

This rule (commonly called *Simpson's Rule*), though not really invented by that mathematician, is very valuable, because it can be applied to irregular figures, as II., III., IV., V., VI.

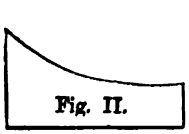


Fig. II.

Fig. III.

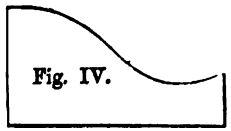


Fig. IV.

In fig. VI. the rule is applied as before, but the first and last ordinates = O, and therefore are not reckoned at all in the calculation; in fig. V., also, the first and last ordinates vanish, and the other ordinates extend across the whole figure, that is, on both sides of the central line.

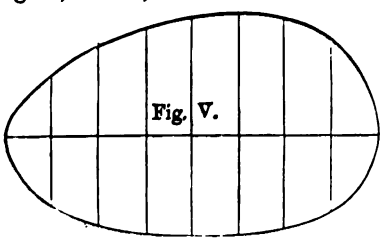


Fig. V.

Fig. VI.

In adopting this method it is advisable to have as many ordinates as possible, since the rule being only approximate, and based upon the supposition that the errors which it introduces will to some extent counterbalance each other, the greater the number of ordinates the nearer the approach to absolute accuracy.

Exercises (U).

Apply the rule of equidistant ordinates to find the areas of the following figures:—

1. The ordinates are 0, 20, 32, 36, 32, 20, and 0 yards, and the common distance 2 yards.
2. The ordinates are 0, 12·5, 16, 13·5, 8, 2·5, 0 yards, and the common distance 2 yards.
3. The ordinates are 0, 3, 5, 7, 5, 7, 9 yards, and the common distance is 1 yard.
4. The ordinates are 4·25, 6·35, 7·35, 8·55, 9·15, 11·35 yards, and the common distance 3 yards.

MISCELLANEOUS EXAMPLES.

1. A pyramid on a square base has each of its eight edges 100 yds. long; what is the length of the edge of a cube which is exactly equal to it in volume?

2. Find the cubical content of a room which is 24 ft. 9 in. long, 18 ft. 4 in. broad, and 10 ft. 8 in. high.

3. A piece of gold leaf is $\frac{1}{1000}$ of an inch in thickness; how much solid gold is there in enough of this to cover 63 square feet?

4. Find the volume of a cylinder whose radius is 8 ft., and height 80 ft.

5. How many square yards are there in $\frac{1}{37}$ of a square mile?

6. Find the area of a triangular field whose sides measure 988, 900, and 848 links.

7. The area of a triangular field is 6 ac. 2 ro. 8 po., and its perpendicular measures 413 links; what will be the cost of erecting a fence along its base at five shillings per rod?

8. A room is 57 ft. long and 18 ft. 9 in. broad; what length of carpet $\frac{3}{4}$ yd. wide will be required to cover the floor?

9. How many square feet of paving will be required for a regular pentagonal court-yard, each of whose sides measures 92·5 ft., and the perpendicular is 63 ft. 8 in.?

10. The sides of a triangular field are 1200, 1000, and 800 links, and it is required to erect a straight line of fencing parallel to the longest side which shall cut off a trapezoidal piece containing 1 ac. 2 ro. 16 po.; find the dimensions of the triangular piece which remains.

11. An upright staff 5 ft. long casts a shadow of 4 ft.; find the height of a church spire whose shadow is 92 ft.

12. The diagonal of a cubical block of wood is 2 ft. 6 in.; find its volume.

13. A panel measuring 2 ft. 8 in. by 1 ft. 6 in. costs 29s. per square foot; what would be the entire cost of four such panels?

14. Find the hypothenuse and area of a right-angled triangle whose base is 168 yds., and perpendicular 425 yds.

15. Find the area of a triangle whose sides measure 277, 315, and 304 yds.

16. The diameter of a sphere is 2 ft. 9 in.; find the area of its surface.

17. A cylindrical boiler is 4 ft. deep, and its diameter is 3 ft.; how many gallons will it hold?

18. The diameter of a circular grass-lawn is 42 yds.; what will be the cost of putting a wire fence round it at 1s. 6d. per yard?

19. A piece of round timber is 14 yds. long, and its quarter girt is 15½ in.; find its solidity.

20. Three squares contain respectively 36, 64, and 576 sq. ft.; find the length of a side of that square which has an area equal to four times their sum.

21. A mirror 5 ft. square is to be cut into the largest possible circular mirror; what is the area which will be wasted?

22. A triangular field contains exactly one acre of land, and its perpendicular measures 40 yds.; find the length of its base.

23. The area of an equilateral triangle is 11·2626 sq. ft.; find its side.

24. A cylinder is made of metal one inch thick, its inner diameter is 3 ft. 9 in.; find its outer circumference, and the area of the piston which works in it.

25. What would be the cost of cleaning and polishing a marble column 20 ft. high, and whose diameter is 2 ft., at 5s. per square yard?

26. The convex surface of a cone is 667·6 sq. ft., and the diameter of its base 8·5 ft.; find its slant height.

27. What would be the cost of staining and varnishing a wooden partition 173 ft. 10 in. long and 10 ft. 7 in. high, at 6s. per square?

28. Calculate the area of a field from the following notes :—

	to B	
	1020	
to G 350	705	
	580	70 to F
	435	320 to E
to D 50	410	
	130	470 to C
	from A	

29. The three sides of the base of a triangular pyramid are 5, 6, and 7 yds., and its volume 71·036 cub. yds.; find its height.

30. Find the area of the ring contained between two concentric circles whose radii are 10 ft. and 16 ft.

31. The radius of a circular lawn is 175 ft.; what will be the cost of returfing it at 3d per sq. yd.?

32. The diagonal AC of a four-sided field is 967 links, and its sides are AB, 751 links; BC, 821 links; CD, 548 links; DA, 715 links; find its area.

33. The diagonal AC of a four-sided field is 800 links, and its sides are AB, 1000 links; BC, 600 links; CD, 205 links; DA, 700 links; find its area.

34. A trapezoid contains 132 sq. yds., and its parallel sides are 45 ft. and 50 ft.; find the perpendicular distance between them.

35. It is required to gravel a triangular yard, one side of which measures 122 ft., and the perpendiclar on it from the opposite angle 97 ft.; the cost of the gravel will be 1s. per sq. yd., and of the labour 1½d. per sq. yd.; find the total cost.

36. A vertical pole 5 ft. high casts a shadow of 7 ft.; required the length of the shadow cast by a chimney 46 yds. high.

37. The diameters of a cylindric ring are 14 ft. and 8 ft.; find its volume.

38. Two squares have sides of 187 ft. and 84 ft.; find the perimeter of the square which is equal in area to four of each of such squares added together.

39. What length of wire ·16 of an in. in diameter can be made out of 2 cubic in. of copper?

40. If a sphere and a cube have the same surface, how much greater is the volume of the sphere than that of the cube?

41. A rectangular field is 2500 links long, and 160 links broad; it is required to cut off a part of it to contain 1 ac. 2 ro. 16 po. by a line parallel to one of its ends; find the length of the part cut off.

42. A circle contains 49·73594 sq. yds.; find its circumference.

43. A court-yard in the form of a regular heptagon contains 1453·56 sq. yds.; find the length of each of its sides.

44. A rectangular cistern is 3 ft. 2 in. long, 2 ft. 8 in. broad, and 2 ft. 6 in. deep, in inside measurement; find the expense of lining it with lead at 4d. per lb., at $3\frac{1}{2}$ lb. weight to the square foot.

45. It requires 14 turns of the handle to raise a bucket from the bottom of a well, and the axle on which the chain is coiled is $10\frac{1}{2}$ in. in diameter; how deep is the well?

46. A street is 42 ft. wide, and the foot of a ladder 54 ft. long is placed 16 ft. from one side of it; to what heights will the ladder reach when leaning against the walls on each side of the street?

47. A ladder 42 ft. long stands on the ground 11 ft. from the wall of a house, against which it leans; if the foot of the ladder be drawn 2 ft. outwards, how far will the top of the ladder descend?

48. How many cubic ft. of earth are thrown out in sinking a well 44 ft. deep, and 4 ft. 6 in. in diameter?

49. The cisterns on the top of a house are supplied with water from a force pump by pipes, whose internal diameter is 2 in., and total length 348 ft.; how many gallons of water have to be pumped up before the water begins to run into the cistern?

50. A gallon of paint will cover 100 sq. yds. of wood work, and costs 7s., and weighs $77\frac{1}{2}$ lb.; what will be the cost of painting a number of wooden partitions 8 ft. high, and, altogether, 570 ft. in length, adding on for the expense of labour, $\frac{1}{4}$d. per square foot?

51. The fore and hind wheels of a carriage are, respectively, 21 in. and 32 in. in diameter; how many times does each turn round in a distance of $1\frac{1}{2}$ miles?

52. A globe 19 in. in diameter weighs 73 lb.; what will be the weight of a globe made of the same material, and whose diameter is 38 in.?

53. How many times greater must be the diameter of a circular grass lawn which contains exactly 2 ac., than that of another circular grass lawn which contains $\frac{1}{2}$ ac.?

54. A rectangular cistern is $4\frac{1}{2}$ ft. long, and $2\frac{1}{4}$ ft. wide; to what depth will 120 gallons of water fill it?

55. How many cubic ft. of gas may be contained in a cylindrical gasometer 54 ft. high, and with diameter 60 ft.?

56. A cistern is 15 ft. long, and 10 ft. broad, and the cost of lining it with lead (lid not included), at 1s. 9d. per sq. yd. is £6, 4s.; find its depth.

57. A square yard of paper is cut up into smaller squares, with sides of $\frac{3}{4}$ in.; how many?

58. A right angled triangle, of which the sides are 7 ft. and 11 ft., is made to revolve on its hypothenuse; find the volumes of the two cones thus formed.

59. Two globes weigh respectively 16 lb. and 25 lb., and the weights of a cubit in. of the materials of which they are made are respectively as 5 to 4; compare the diameters of the globes.

60. A cylinder, 5 ft. in diameter and 10 ft. high, is closed at each end by a hemisphere; find the volume of the whole figure.

61. A cylinder, 8 yds. long and with radius 2 ft., is closed at each end by a hemisphere; find the cost of painting it at one halfpenny per square foot.

62. The volume of a cube is 178 cubic ft.; find the length of its diagonal.

63. How many sq. yds. are there in a triangle whose sides are 12 yds., 8 yds., and 16 yds.?

64. What is the area of a board 30 ft. long, the breadth at one end being 3 ft. 9 in., and at the other 1 ft. 9 in.?

65. Find the area of a field in the form of a trapezoid, of which the parallel sides are 250 yds. and 150 yds., and the perpendicular distance between them 80 yds.

66. Find the area of a field in the form of a trapezoid of which the parallel sides are 100 yds. and 200 yds., and the perpendicular distance between them 50 yds.

ANSWERS TO EXERCISES.

(A).

1. 424 nls.
2. 246834 ft.
3. 488 ac. 3 ro. 35 po.
4. 75174 po.
5. 271 yds. 3 qrs.
6. 2 miles 2 fur. 147 yds.
7. 11400 ells.
8. 58690½ yds.
9. 17 ac. 2 ro.
10. 78254 po.
11. 8 cubic yds. 10 ft. 728 in.
12. 24478 po.
13. 125215½ yds.
14. 110113½ ft.
15. 3470 in.
16. 3924241 in.
17. 126 hrs. 49$\frac{5}{11}$ min.
18. 39138 ac. 0 ro. 14 po. 12½ yds. 7 ft. 72 in.
19. 2 ac. 1 ro. 32 po. 11 yds. 8 ft. 22 in.
20. 13 cubic yds. 26 ft. 143 in.

(B).

1. 77° 22′ 22″.
2. 30° 60° 90°.
3. 45° 45° 90°.
4. 84 yds. 7½ in.
5. 134·01 ft.
6. 22 ft. 5 in.
7. 94 ft. 10 in.
8. 64 ft. 10 in.
9. 137 ft.
10. 104 ft. 10 in.
11. 13 ft. 10 in.
12. 528 yds.
13. 44 ft.
14. 5160 yds.
15. 1414·2 ft.
16. 17 ft. 3 in.
17. 60 ft.
18. £282.
19. £31, 1s. 8$\frac{3}{4}$d.
20. £33, 16s. 8d.
21. 36 ft.
22. 76·123 ft.
23. 42 ft. 5 in.
24. 319·99 ft.

(C).

1. 37$\frac{1}{18}$ sq. yds.
2. 29 sq. ft. 4 in.
3. 12 ft.
4. 45 ft.
5. 5 sq. yds. 2 ft. 76 in.
6. 2 sq. yds. 6 ft.
7. 6 ft.
8. £16, 0s. 7d.
9. 11 ac. 0 ro. 4 po.
10. 173·92 yds.
11. 6050 sq. yds. and 61S272 sq. yds.
12. 35 sq. yds. 5½ ft.
13. £1237, 10s.
14. 69½ yds.
15. 285·72 sq. yds.
16. 7 ac. 2 ro. 13·8 po.

(D).

1. 57 ac. 3 ro. 20 po.
2. 295$\frac{83}{88}$ yds.
3. 57$\frac{6}{14}$ sq. yds.
4. 660 ft.
5. £4, 15s. 7d.
6. 135·16 sq. ft.

(E).

1. 104 sq. ft. 88 in.
2. 56 ac. 2 ro. 31 po.
3. 205.09 sq. ft.
4. 4·6875 sq. ft.
5. 34 sq. ft. 63 in.
6. 12 ac. 2 ro. 4 po.
7. 20 sq. yds. 6 ft.
8. 292 sq. yds. 1 ft. 115½ in.
9. 5 ac. 0 ro. 36 po.

10. 5 ac. 3 ro. 23·5 po.
11. 235·12 sq. yds.
12. 3 ac. 2 ro. 21 po.
13. 2 ac. 2 ro. 16·5 po.
14. 427 sq. yds. 5 ft. 114 in.
15. 134 sq. yds. 64 in.
16. 141 sq. yds. 6 ft. 8 in.
17. 9 ac. 24 po.
18. 8 ac. 1 po.
19. 8 ac. 3 ro. $20\frac{4}{11}$ po.
20. 7 ac. 3 ro. $7\frac{3}{4}$ po.
21. $15\frac{7}{11}$ ft.
22. 3 ac. 0 ro. 13·75 po.
23. 690 sq yds.
24. 1026 sq. yds.
25. 19536 sq. yds.
26. 14760 sq. yds.

(F).

1. 13 ac. 2 ro. 16 po.
2. 152·075 sq. ft.
3. $16\frac{7}{45}$ sq. ft.
4. $167\frac{9}{32}$ sq. ft.
5. 86·434 sq. yds.
6. 7 ac. 2 ro. $13\frac{1}{2}$ po.
7. 229 sq. yds.
8. $871\frac{1}{2}$ sq. yds.
9. 52330·3429 sq. yds.
10. 2161 sq. yds. 0 ft. 9 in.
11. 123 ac. 0 ro. 11·8656 po.
12. $12\frac{1}{4}$ chains, $8\frac{3}{4}$ chains.

(G).

1. 672 sq. ft. 66·204 in.
2. £5, 9s. $0\frac{1}{2}$d.
3. 1231 sq. yds. 1 ft. 18·36 in.
4. 3 ft. 5·9802 in. and 4 ft. 3·89 in.
5. 58888·7375 sq. in.
6. $30\frac{5}{8}$ sq. ft.
7. 69 ac. 2 ro. 4·6 po.
8. 43·0125 sq. yds.
9. 333 ac. 2·9 ro.
10. $4301\frac{1}{4}$ sq. ft.
11. 6193·8 sq. ft.
12. 123 sq. yds. ·936 ft.
13. 387 sq. yds. 1 ft. 18 in.
14. 12 ac. 0 ro. 11·4 po.

(H).

1. 87·96452 yds.
2. 376·99 ft.
3. 76·183 ft.
4. 24856·26 miles.
5. 11 ft. 9·37 in.
6. 2774715·1198 miles.
7. 24127·4122 miles.
8. 25000·7732 miles.
9. 1517·3879 ft.
10. 395·8403 ft.
11. 45·8366 miles.
12. 7·87817 ft.
13. 3·02394 ft.
14. 2180·42456 ft.
15. 56·34089 ft.
16. 397·88769 miles.
17. 3·50109 ft.
18. 20·37121 ft.
19. 153·7438 ft.
20. $17\frac{1}{2}$ in.
21. 153·9384 sq. links.
22. 7854 sq. ft.
23. 6126·752 sq. yds.
24. 1 ac.
25. 468·2347 sq. yds.
26. 1·12045 yds.
27. 4·6 times.
28. 4398·72206 sq. yds.
29. 236·599 yds.
30. £512, 13s. $7\frac{1}{2}$d.
31. 14·14213 ft.
32. 436 tons, 4 cwt. 2 qrs. 20 lb.
33. 38·0942034 yds.
34. 201·0624 sq. ft.
35. 5 ac. 1 ro. 15 po.
36. 960 times.

(I).

1. 1 sq. yd. 7·964586 ft.
2. 100 sq. ft.
3. 89·38 sq. yds.
4. 13·6587 sq. yds.
5. 1687·8158 sq. yds.
6. 13·544 ft.
7. 17·7498 sq. ft.
8. 100 sq. ft.
9. 140 sq. yds. 97 in.
10. 593·9568 sq. ft.

(J).

1. 47·124 sq. ft.
2. 32·9868 sq. ft.
3. 667½ sq. yds.
4. 1016 stakes.
5. 2748·99 sq. yds.
6. 1 ac. 396 sq. yds.
7. £48, 7s. 7d.
8. 150·7968 sq. in.

(K).

1. 15·625 cub. ft.
2. 394 cub. yds. 10 ft.
3. 421·875 cub. ft.
4. 296 cub. ft. 3 in
5. 274·625 cub. ft.
6. 2072·67 cub. ft.
7. 22 cub. ft. 8 in.
8. 20 cub. ft. 9 in.
9. $37\frac{1}{27}$ cub. ft.
10. 133·cub. yds. 14 ft. 1 in.
11. 1157 cub. ft. 90 in.
12. $394\frac{19}{27}$ cub. ft.
13. $204\frac{1}{4}$ sq. ft.
14. $630\frac{8}{9}$ sq. ft.
15. $234\frac{3}{8}$ sq. ft.
16. 198 sq. ft. 4 in.
17. 14210 cub. yds.
18. 108·44 cub. yds.
19. 677 cub. ft. 4 in.
20. 1738½ cub. yds.
21. 45 cub. ft. 11 in.
22. 201·25 cub. yds.
23. 17066 cub. ft. 1152 in.
24. 4 cub. yds. 21·5 ft.
25. 18 ft.
26. 1 ft. 1·7 in.
27. £2, 15s. 11½d.
28. £3, 0s. 9d.
29. 3 ft. 2·5 in.
30. 5834 gals.
31. 19¼ in.
32. 6 yds.

(L).

1. 183·75 cub. ft.
2. 19313·7084 cub. ft.
3. 8 cub. yds. 24 ft.
4. 64·0625 cub. ft.
5. 6 cub. yds. 4·27687 ft.
6. 318 cub. yds. 16·387 ft.
7. £8, 7s. 8½d.
8. 205·00447 cub. ft.
9. 16·002 ft.
10. 23·99 ft.
11. 5·501 ft.
12. 976·577 cub. ft.

(M).

1. 51 cub. ft. 972 in.
2. 5 cub. ft. 199 in.
3. 27 sq. yds. 8 ft. 61 in.
4. 93 sq. yds. 7 ft. 36 in.
5. 5 cub. yds. 9 ft. 648 in.
6. 72 cub. yds. 20 ft. 1398 in.
7. £26, 14s.
8. 6 cub. yds. 7 ft. 1204 in.

(N).

1. 311·769 cub. ft.
2. 275·276 cub. ft.
3. 46·4456 sq. ft.
4. 2 tons, 1 cwt. 0 qr. 8·744 lb.
5. 32 cub. ft. 1404 in.
6. 737·22 cub. ft.

(O).

1. 70·68 cub. ft.
2. 265·14 cub. ft.
3. £30, 0s. 0½d.
4. 26 times.
5. 12 ft. 4 in.
6. 202·65 cub. ft.
7. £7, 14s. 6d.
8. £19, 10s. 1½d.
9. 6½d.

(P).

1. £7, 19s. 8½d.
2. $449\frac{17}{168}$ cub. in.
3. $862\frac{2}{7}$ cub. in.
4. 11·237 in.
5. 22·32 in.

6. 23·144 ft.
7. 4419·6 sq. ft.
8. 24124·94 miles, and 185186000·2281 sq. miles.
9. 5·9 ft.
10. 5·09 sq. yds.
11. 321·674 cub. in.
12. 122·835 cub. ft.
13. 23·767 sq. ft.
14. 4 ft. 3 in.
15. 5·46 ft.
16. 1404928 times.

(Q).

1. 156 cub. ft.
2. 37·83 cub. ft.
3. 30·58 cub. ft.
4. 7s. 4d.
5. 82·68 cub. ft.
6. 457·5 cub. ft.
7. 24·6675 cub. ft.
8. 9·315 cub. ft.

(R).

1. 7·036 cub. ft.
2. 25·52 cub. ft.
3. 5·05 cub. ft.
4. 370·324 cub. ft.
5. 259·7 cub. ft.
6. 183·78 cub. ft. and 98·96 cub. ft.

(S).

1. £50, 1s. 5½d.
2. £15, 15s. 0d.
3. 16⅔ cub. ft.
4. £3, 15s. 7½d.
5. 54·1972 gals. nearly.

(T).

1. 5 ac. 0 ro. 5·6 po.
2. 15 ac. 0 ro. 26 po.
3. 4 ac. 3 ro. 38·16 po.
4. 1 ac. 0 ro. 16·7 po.

(U).

1. 161⅓ sq. yds.
2. 101⅓ sq. yds.
3. 32⅓ sq. yds.
4. 153·6 sq. yds.

Miscellaneous Examples.

1. 61·77 yds.
2. 4840 cub. ft.
3. 9 cub. in.
4. 16084·9408 cub. ft.
5. 96800 sq. yds.
6. 3 ac. 2 ro. 9 po.
7. £31, 14s. 4½d.
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